PC 20 Section 6.3– Adding and Subtracting Rational Expressions

PreCalculus 20 • Saskatchewan Curriculum • McGraw-Hill Ryerson PreCalculus 11, Section 6.3

1. The key idea: use a common denominator

A rational expression is a fraction whose numerator and/or denominator contains a polynomial. ***Adding and subtracting rational expressions follows the same principle as adding and subtracting numerical fractions: you must have a common denominator before the numerators can be combined.***

$$\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$$

Remember: You may add or subtract the numerators only after the expressions have equivalent, common denominators. ***Do not add the denominators.***

2. When the denominators are already the same

Keep the common denominator and combine the numerators.

$$\frac{2x + 1}{x - 3} + \frac{x - 5}{x - 3} = \frac{2x + 1 + x - 5}{x - 3}$$

$$= \frac{3x - 4}{x - 3}$$

The denominator is not added. The numerator is simplified by collecting like terms.

$$\frac{5x - 2}{x + 4} - \frac{2x + 7}{x + 4} = \frac{5x - 2 - (\mathbf{2x + 7})}{x + 4}$$

$$= \frac{5x - 2 - 2x - 7}{x + 4} = \frac{3x - 9}{x + 4}$$

Use *brackets* around a numerator when *subtracting* so that every sign is handled correctly.

Sign reminder: Subtract the entire second numerator. ***Distribute the negative sign*** to every term inside its brackets.

3. Non-permissible values

A denominator cannot equal zero. Values that make an original denominator zero are called ***non-permissible values*** (NPVs). State them even if a factor later cancels.

$$\frac{x + 2}{x - 4} + \frac{3}{x - 4}$$

The common denominator is x − 4, so x − 4 ≠ 0 and x ≠ 4. The restriction comes from the original denominators.

4. Different denominators: find the LCD

When denominators differ, factor them completely first. The lowest common denominator (LCD) contains each distinct factor the greatest number of times it appears in any denominator.

Example 1: Add the following rational expressions.

$$\frac{2}{x+3}+\frac{1}{x-2}$$

Step 1: The denominators have no common factor, so the LCD is (x + 3)(x − 2).

$$\frac{2(x-2)}{(x+3)(x-2)}+\frac{1(x+3)}{(x+3)(x-2)}$$

Step 2: Expand the numerators, then combine over the LCD.

$$\frac{2x-4+x+3}{(x+3)(x-2)}$$

$$=\frac{3x-1}{(x+3)(x-2)}$$

Restrictions: x ≠ −3 and x ≠ 2.

Equivalent fractions: Multiply the numerator and denominator by the missing factor. This changes the form, but not the value, of the rational expression.

5. Factor first when possible

Factoring can reveal common factors and make the LCD easier to identify. Do not expand a denominator unnecessarily.

Example 2: Subtract. Factor the quadratic denominator first.

$$\frac{x}{x^2-9}-\frac{1}{x+3}$$

Step 1: Factor x² − 9 = (x − 3)(x + 3). The LCD is (x − 3)(x + 3).

$$\frac{x}{(x-3)(x+3)}-\frac{x-3}{(x+3)(x-3)}$$

Step 2: Subtract the complete second numerator.

$$\frac{x-(x-3)}{(x-3)(x+3)}$$

$$=\frac{3}{(x-3)(x+3)}$$

Restrictions: x ≠ 3 and x ≠ −3. The numerator simplifies to 3; no denominator factor cancels.

Final check: Factor the denominators, identify the LCD, rewrite each fraction, combine the numerators, simplify, and state all original restrictions.

6. A reliable procedure

1. **Factor** each denominator completely.
2. **Identify** the LCD by including every required factor.
3. **Rewrite** each rational expression using the LCD.
4. **Combine** the numerators; use brackets when subtracting.
5. **Simplify** the numerator and factor if possible.
6. **State** all non-permissible values from the original denominators.

7. Common errors to avoid

- Adding or subtracting denominators instead of finding a common denominator.
- Forgetting to multiply the numerator by the same missing factor used in the denominator.
- Forgetting brackets or failing to distribute a negative sign when subtracting.
- Cancelling terms across addition or subtraction. Only common factors may be cancelled.
- Leaving out a non-permissible value because a factor disappears during simplification.

8. Try these practice problems

Show your work. Factor denominators where appropriate, use a common denominator, simplify, and state all non-permissible values.

1. $\frac{3}{x+1} + \frac{2}{x+1}$

2. $\frac{2}{x} + \frac{3}{x+2}$

3. $\frac{4x-1}{x^2-25} - \frac{x+2}{x+5}$

4. Word problem: A student can complete one portion of a landscaping job in x hours. A second student can complete one portion in x + 2 hours. Their work rates are therefore 1/x and 1/(x + 2) jobs per hour. Write and simplify an expression for their combined rate. State the values of x that are not permitted.

Homework: