6.2 – Multiplying and Dividing Rational Expressions

MULTIPLYING RATIONAL EXPRESSIONS

1. Review: Multiplying Fractions

Multiplying rational expressions is an extension of multiplying ordinary fractions.

First, remember the fraction rules: ***multiply the numerators together*** and the ***denominators together***. ***Then simplify*** by dividing the numerator and denominator by any common factors.

$$\text{Example: } \frac{2}{3} \times \frac{9}{8} = \frac{2 \times 9}{3 \times 8} = \frac{18}{24} = \frac{3}{4}$$

You can often *simplify before multiplying*. For example, 2 and 8 share a factor of 2, and 9 and 3 share a factor of 3:

$$\frac{2}{3} \times \frac{9}{8} = \frac{1}{1} \times \frac{3}{4} = \frac{3}{4}$$

Tip: Cancelling before multiplying *keeps the numbers smaller*. You are dividing the numerator and denominator by the same non-zero factor, so the value of the expression does not change.

2. Multiplying Rational Expressions

Because of the multiplying rules for fractions, multiplying rational expressions is the same as simplifying

Simplifying Rational Expression Example:

$$\frac{2(x-3)(x+3)}{4x(x-3)}$$
$$= \frac{x+3}{2x}$$

Multiplication of Rational Expressions Example

$$\frac{2(x\ -\ 3)}{4x} \times \frac{x\ +\ 3}{x-3}$$
$$= \frac{2(x-3)(x+3)}{4x(x-3)}$$
$$= \frac{x+3}{2x}$$

Think of the entire multiplication as one large rational expression. All factors can be combined into one fraction. Factors in the numerator and denominator ACROSS the MULTIPLICATION SIGN can also be divided out!

1. Factor each numerator and denominator completely.
2. Identify non-permissible values from the ORIGINAL denominators.
3. TREAT MULTIPLICATIONS AS ONE LARGE RATIONAL.
4. Divide out common factors in the numerator and denominator.
5. Multiply the remaining factors and simplify.

You try $\frac{x^2-4}{5x} \times \frac{3x}{x+2}$

Important: *Cancel complete factors, not terms. For example, (x + 2) is a factor and can be cancelled when it appears in both numerator and denominator. You cannot cancel the x in x + 2 because x is only one term in a sum.*

PreCalculus 20 6.2 MM Notes Name: _____

DIVIDING RATIONAL EXPRESSIONS

4. Review: Dividing Fractions

To divide ordinary fractions, use KEEP – CHANGE – FLIP and MULTIPLY:

1. ***Keep the first fraction*** exactly as it is.
2. ***Change the division sign (÷) to multiplication (×).***
3. ***Flip the second fraction*** (the divisor) to its reciprocal.

Example: $\frac{2}{3} \div \frac{4}{9} = \frac{2}{3} \times \frac{9}{4} = \frac{18}{12} = \frac{3}{2}$

OR Divide out common factors before multiplying:

$$\frac{2}{3} \div \frac{4}{9} = \frac{2}{3} \times \frac{9}{4} = \frac{\cancel{2}\,1}{\cancel{3}\,1} \times \frac{\cancel{9}\,3}{\cancel{4}\,2} = \frac{3}{2}$$

Remember: Only the second fraction is flipped. Do not flip the first fraction.

5. Dividing Rational Expressions

We can easily turn DIVISION OF RATIONALS into MULTIPLICATION OF RATIONALS.
Use the same pattern with rational expressions: KEEP – CHANGE – FLIP – FACTOR – CANCEL – MULTIPLY.
Example 2

$$\frac{2(x-1)}{3} \div \frac{(x-1)(x+1)}{5}$$

Keep and change: $\frac{2(x-1)}{3} \times$

Flip the divisor: $\frac{2(x-1)}{3} \times \frac{5}{(x-1)(x+1)}$

Divide out $(x-1)$, then multiply: $\frac{2 \times 5}{3(x+1)} = \frac{10}{3(x+1)}$

Can't equal zero

$\frac{2(x-1)}{3} \div \frac{(x-1)(x+1)}{5}$

Final answer: $\frac{10}{3(x+1)}$ *x ≠ 1, −1*

Why are both values restricted? Notice that ***WE CANNOT DIVIDE BY ZERO, and x = 1 and −1 makes the second term equal to zero***. A divisor cannot equal zero.

Example:

Simplify:

$$\frac{x^2-9}{2x} \div \frac{x+3}{4}$$ *Note restrictions* *x ≠ 0, −3*

Keep – Change – Flip, then factor:

$$= \frac{x^2-9}{2x} \times \frac{4}{x+3} = \frac{(x-3)(x+3)}{2x} \times \frac{4}{x+3}$$

Cancel the common factor (x + 3), then simplify 4/2:

$$= \boxed{\frac{2(x-3)}{x} \quad x \neq 0, -3}$$

***The original expression requires x ≠ 0 because of the first denominator, and x ≠ −3 because the divisor would equal zero.*

7. Quick Checklist: Avoid Common Mistakes

- Show multiplying and dividing as multiplication
- Find non-permissible values from the original expression, before dividing out.
- Factor completely before dividing out.
- Divide out common factors, not separate terms in a sum or difference.
- When dividing rationals, flip ONLY the second rational expression.
- After flipping, multiply and simplify as usual.
- Every original denominator must be non-zero, and the divisor itself must also be non-zero.

Homework: