PC 20 – 6.1 Rational Expressions Introduction

Section 6.1: Rational Expressions | McGraw-Hill Ryerson PreCalculus 11

1. What Is a Rational Expression?

A rational expression is an algebraic fraction in which the ***numerator and denominator are both polynomials.*** It has the general form $\frac{p}{q}$, where p and q represent polynomials (and $q \neq 0$). Rational expressions are **similar to regular fractions, except they contain algebraic variables.**

Examples: $\frac{x^2-2}{x^2+3x+8}$ $\frac{3}{x+6}$ $\frac{-x^2}{x^2-1}$ $\frac{3x^2-2x}{x^2-8}$

2. Why Can't the Denominator Equal Zero?

Division by zero is undefined in mathematics. This means the denominator of a rational expression can never equal zero. A ***non-permissible value*** is a specific *value of the variable* that ***causes the denominator to evaluate to zero.***

Example: Consider the expression $\frac{6}{x-3}$ If $x = 3$, then $x - 3 = 0$. Therefore, $x = 3$ is non-permissible, and the restriction is stated as $\boldsymbol{x \neq 3}$

3. Identifying Non-Permissible Values

To find non-permissible values, analyze the original denominator using the following process:

1. Factor the denominator fully.
2. Set the denominator equal to zero.
3. Set each factor = 0 then solve for the variable.
4. State the value or values that are restricted from the domain.

Example: Find restrictions for $\frac{x+4}{x^2-9}$

1. Factor the denominator fully $x^2 - 9 \Rightarrow (x-3)(x+3)$
2. Set the denominator equal to zero $(x-3)(x+3) = 0$
3. Set each factor = 0 then solve for the variable $(x-3) = 0,\ (x+3) = 0$
   $x = 3 \text{ or } x = -3$
4. State the value or values that are restricted from the domain $\boldsymbol{x \neq 3,\ x \neq -3\ (\text{or } x \neq \pm 3)}$

4. Simplifying Rational Expressions

A rational expression is simplified by ***dividing out common factors shared by the numerator and denominator.***

Worked Example: Simplify $\frac{x^2 - 9}{x^2 + 5x + 6}$

Step 1: Factor numerator and denominator completely

$= \frac{(x - 3)(x + 3)}{(x + 2)(x + 3)}$

Step 2: Note the non-permissible values from denominator

Restrictions: $x \neq -2,\ x \neq -3$

Step 3: Divide out the common factor *(x + 3)*

$= \frac{(x - 3)}{(x + 2)}$

Step 3: State the final simplified answer with the restrictions

$= \frac{(x - 3)}{(x + 2)}$ *Restrictions:* $x \neq -2,\ x \neq -3$

KEY SUMMARY

1. **Factor First**: Always factor the numerator and denominator completely before attempting to simplify.
2. **Identify Restrictions Early**: Non-permissible values must be determined from the ORIGINAL denominator before any factors are cancelled.

Examples: $= \frac{(x - 3)(x + 3)}{(x + 2)(x + 3)}$

$= \frac{(x - 3)}{(x + 2)}$

*if you note restrictions ***after*** simplification you may miss $x \neq -3$

3. **Cancel Factors, Not Terms**: You may only divide out common multiplicative factors, never individual terms.

Examples: $\frac{(x - 3)\cancel{(x + 3)}}{(x + 2)\cancel{(x + 3)}} = \frac{(x - 3)}{(x + 2)}$ this is correct, you are dividing out whole factors,

$\frac{\cancel{x^2} - 9}{\cancel{x^2} + 5x + 6} \neq \frac{-9}{5x + 6}$ this is incorrect – you must not 'divide' out single terms, they must be common *factors* (expressions or terms MULTIPLIED together!)

Homework: