PC 20 5.3 - Solving Radical Equations

1. ***Definition***: A **radical equation** is an algebraic equation where the ***variable is located underneath a radical symbol.***

Example: Solve $\sqrt{(x+2)} = 4$

$\sqrt{(x+2)}^2 = 4^2$

$x + 2 = 16$

$x = 14$

Check: $\sqrt{(14+2)} = 4$

$\sqrt{16} = 4$

$4 = 4$

Step 1 - Isolation: Always begin by **isolating** one of the radical terms on one side of the equation

Step 2 - Elimination: To **eliminate a root**, raise both sides of the equation to the ***power of the index***

Step 3 – Solve for the known variable

Step 4 – Check your solutions

Always **substitute** your final answers back into the original equation to check for validity and catch *extraneous roots*.

2. ***Extraneous Roots:*** Raising both sides to an even power can introduce an extraneous root—"***a solution that solves the manipulated equation but fails the original.***"

Example: Solve the following equation using the above steps

$n - \sqrt{5-n} = -7$

$-\sqrt{5-n} = -7 - n$

$\sqrt{5-n} = 7 + n$

$(\sqrt{5-n})^2 = (7+n)^2$

$5 - n = 49 + 14n + n^2$

$0 = 44 + 15n + n^2$

At this point you may use any means to solve the quadratic

Method 1: Factoring

$(n+11)(n+4) = 0$

$n = -11, -4$

Method 2: Quadratic Formula

$$n = \frac{-15 \mp \sqrt{15^2 - 4(1)(44)}}{2(1)}$$

$n = -11, -4$

Check these answers!

Check: $-11 - \sqrt{5-(-11)} = -7$

$-11 - 4 = -7$

$-16 \neq -7$

$-4 - \sqrt{5-(-4)} = -7$

$-4 - 3 = -7$

$-7 = -7$

Final answer… $n = -4$

$n = -11$ does not check out, this is a case where we have an ***extraneous root*** that must be rejected

3. ***Domain Limits & Restrictions:***

When determining **restrictions**, keep these rules in mind:

a. **Radicands** (expressions under even roots) must be **non-negative** whenever the **index** is an *even number*, and

b. **Denominators** cannot equal zero.

Example: State the restrictions and solve the following:

$\mathbf{5 + \sqrt{2x - 1} = 12}$ Restrictions on the variable: $2x - 1 \geq 0$ *so* $x \geq \frac{1}{2}$

$\sqrt{2x - 1} = 7$

$2x - 1 = 49$

$2x = 50$

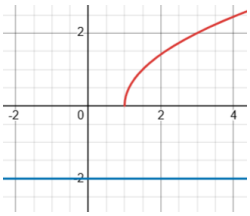
$x = 25$

Special note! If an ***isolated 'even root' radical is equal to a negative number*** There are ***NO solutions!!***

Example:

$\sqrt{2x - 2} = -2$

*Graphing both sides of the function would produce NO intersection points



4. ***Squaring twice to solve***

Where there are more than one radicals in an equation, isolate one radical, square both sides and then you will have to repeat the process.

Example: Solve $\mathbf{7 + \sqrt{3x} = \sqrt{5x + 4} + 5}$

-Isolate radical
-Square both sides
-Isolate radical
-Square both sides
-Solve.

Word Problem:

The mass, m, in kilograms that a beam with a fixed width and length can support is related to its thickness, t, in cm. The formula is $t = \frac{1}{5}\sqrt{\frac{m}{3}}$ where $m \geq 0$. If a beam is 6 cm thick, what mass can it support?

Your turn: Solve each radical equation. Verify your solutions.

a) $\mathbf{\sqrt{k + 5} = \sqrt{2k - 1}}$

b) $\mathbf{9 + \sqrt{z} = 14}$

Homework: