PC 20 Section 5.2 – Multiplying and Dividing Radicals

1. Multiplying Radicals with Identical Indices

Recall: When multiplying polynomial terms we multiply coefficients together, and then multiply variables together. Eg. $2s \times 3s^2 = 6s^3$

Similarly, when multiplying radicals that have the same index, multiply the **coefficients** (numbers outside the radical) together and multiply the **radicands** (numbers inside the radical) together. Then simplify the result if possible.

Examples:

$\sqrt{5} \times \sqrt{3}$
$= \sqrt{5 \times 3}$
$= \sqrt{15}$

$2\sqrt{6} \times \sqrt{3}$

$4\sqrt{3} \times 7\sqrt{x}$

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$
$$\sqrt{a} \times \sqrt{a} = \sqrt{a^2} = a$$
$$a\sqrt{b} \times c\sqrt{d} = ac\sqrt{bd}$$

Remember: If the index is even, the radicands also must be non-negative real numbers. STATE RESTRICTIONS if there are any.

2. Dividing Radicals with Identical Indices

When dividing radicals with the same index, divide the coefficients and divide the radicands. Simplify the resulting expression whenever possible.

Examples:

$\frac{2\sqrt{6}}{\sqrt{3}}$

$= 2\sqrt{2}$

$\frac{8\sqrt{26}}{2\sqrt{13}}$

$\frac{8\sqrt{3x}}{2\sqrt{x}}$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$
$$\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} \cdot \frac{\sqrt{b}}{\sqrt{b}} = \frac{\sqrt{ab}}{b}$$
$$\frac{a}{\sqrt{b}} = \frac{a}{\sqrt{b}} \cdot \frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$$

Remember:

1. You can only multiply or divide radicals like this when they ***have the same index***.
2. The ***denominator cannot equal zero***.

You try: $\frac{2\sqrt{3}}{10\sqrt{6}}$

3. Multiplying Radical Expressions with Multiple Terms

When multiplying expressions containing more than one term, use the **distributive property** (often called FOIL when multiplying two binomials). Multiply each term in the first expression by every term in the second expression. Then combine like terms and simplify.

Examples: $2\sqrt{6}\,(\sqrt{3} + 7\sqrt{5})$

$= (2\sqrt{6})\,(\sqrt{3}) + (2\sqrt{6})\,(7\sqrt{5})$

$= 2\sqrt{18} + 14\sqrt{30}$

$= 6\sqrt{2} + 14\sqrt{30}$

You try: $x\sqrt{5}\,(\sqrt{5x} + 7\sqrt{11})$

More examples:

$(\sqrt{3}+7\sqrt{10})(\sqrt{5}+\sqrt{2})$
$= \sqrt{15} + \sqrt{6} + 7\sqrt{50} + 7\sqrt{20}$
$= \sqrt{15} + \sqrt{6} + 7\sqrt{25 \times 2} + 7\sqrt{4 \times 5}$
$= \sqrt{15} + \sqrt{6} + 35\sqrt{2} + 14\sqrt{5}$

$(\sqrt{3a}+\sqrt{2})(\sqrt{5}+\sqrt{a})$

4. Rationalizing a Monomial Denominator

To rationalize a denominator means to **remove the radical from the bottom of a fraction**. Multiply both the numerator and denominator by an expression that makes the denominator a rational number (a number without a radical).

Example: Rationalize the denominator

$$\frac{8}{2\sqrt{5}}$$

$$= \frac{8\times\sqrt{5}}{2\sqrt{5}\times\sqrt{5}}$$

$$= \frac{8\sqrt{5}}{2(\sqrt{5})^2}$$

$$= \frac{8\sqrt{5}}{10}$$

$$= \frac{4\sqrt{5}}{5}$$

Notice that $\sqrt{5} \times \sqrt{5} = 5$, *leaving no radical in the denominator.*

$$\frac{7}{2\sqrt[3]{3}}$$

$$= \frac{7\times\sqrt[3]{3}\times\sqrt[3]{3}}{2\times\sqrt[3]{3}\times\sqrt[3]{3}\times\sqrt[3]{3}}$$

$$= \frac{7(\sqrt[3]{3})^2}{2(\sqrt[3]{3})^3}$$

$$= \frac{7(\sqrt[3]{3})^2}{6}$$

You Try! Rationalize the denominator $\frac{3\sqrt{5}}{\sqrt{6}}$

5. Rationalizing a Binomial Denominator

When the denominator contains two terms, one or both of which involve a square root, ***multiply the numerator and denominator by the conjugate of the denominator***.

A ***conjugate is formed by changing the sign*** between the two terms.

- The conjugate of $(\sqrt{5} + 2)$ is $\sqrt{5} - 2)$.
- The conjugate of $(\sqrt{x} - 1)$ is $(\sqrt{x} + 1)$.

**This works because we end up with a difference of squares!*

Homework:

Example: Rationalize the denominator

$$\frac{1}{\sqrt{x}+1}$$