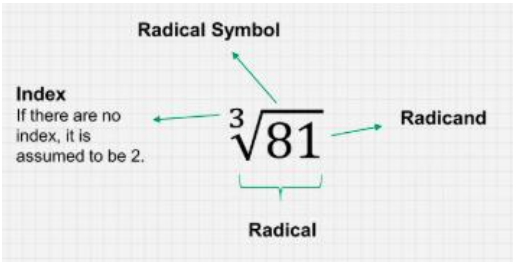
PC 20 — 5.1 Working with Radicals

A radical is an expression involving a root, such as $\sqrt{12}$ or $\sqrt[3]{-14a^3}$. The number written inside is the **radicand**; the small number showing which root to take is the index. If no index is shown, it is 2.



1. Add and subtract like radicals

You can combine 'like terms' into one term when the variable parts are the same. Radicals are 'like radicals' when they have the **same index** and the **same radicand after simplifying**. *Combine only the coefficients*; do **not** add the radicands.

$$3\sqrt{7} + 5\sqrt{7} = 8\sqrt{7}$$

2. Simplify radicals

A radical is in ***simplest form when*** *no fraction remains inside the radical*, the *radical is not in a denominator* and ***the radicand has no removable perfect-power factor***. For square roots, a perfect-power factor is a perfect square factor.

For a square root, identify the largest perfect square factor.

For a cubed root, identify the largest perfect cube factor, etc

Example: Simplify $\sqrt{40}$

$= \sqrt{4 \times 10}$

$= \sqrt{4} \times \sqrt{10}$

$= 2 \times \sqrt{10}$

$= 2\sqrt{10}$

Example 2: Simplify $2\sqrt[3]{16}$

$= 2 \times \sqrt[3]{8} \times \sqrt[3]{2}$

$= 2 \times \mathbf{2} \times \sqrt[3]{2}$

$= 4\sqrt[3]{2}$

You try! Simplify $5\sqrt{27}$

3. Entire and Mixed radicals

Radicals that have a coefficient (multiplier outside the rad. sign) of '1' are entire radicals (the **entire** number is under the rad sign). If the coefficient is anything other than '1', then it is a mixed radical.

Examples:

Mixed	Entire
$3\sqrt{5}$	$\sqrt{45}$
$6b^7\sqrt{3}$	$\sqrt{108b^{14}}$
$a\sqrt[3]{14}$	$\sqrt[3]{14a^3}$

4. Converting Mixed and Entire radicals

Entire → Mixed is the same as simplifying radicals. Examine and simplify ***largest perfect-power factor***

Mixed → Entire In this case we ***apply the index of the radical as a power to the coefficient*** in order to place it under the radical sign.

Mixed		Entire
$3\sqrt{5}$	$\sqrt{(3^2)(5)}$	$\sqrt{45}$
$6b^7\sqrt{3}$	$\sqrt{((6b^7)^2)(3)}$	$\sqrt{108b^{14}}$
$a\sqrt[3]{14}$	$\sqrt[3]{a^3 14}$	$\sqrt[3]{14a^3}$

5. When is a variable radical real?

For an ***even index***, the **radicand must be non-negative**. For an ***odd index***, the **radicand may be any real number.**

Try these (show your work)

1. Simplify: $\sqrt{48} - \sqrt{12}$

2. Simplify: $5\sqrt{48} - 2\sqrt{12} + \sqrt{75}$

3. Order from least to greatest: $\sqrt{45}$, 8, $2\sqrt{12}$

4. State the real-number restriction for $\sqrt{2x+6}$

Word problem:

5. A square garden has area 72 m². Write its exact side length in simplest radical form and give an approximate length to the nearest tenth.

6. Simplify: $\sqrt{4x^6}$

7. Simplify $\sqrt[3]{32a^{10}}$

Homework: