PreCalculus 20 – 4.3 Solving Quadratic Equations by Completing the Square

1. Solving a Squared Binomial Equation

Consider $(x + 3)^2 = 25$. This equation contains a ***variable that is easy to isolate***, so we can take the square root of both sides.

$x + 3 = \pm 5$

$x + 3 = 5$ or $x + 3 = -5$

$x = 2$ or $x = -8$

The symbol ± reminds us to use both the positive and negative square roots. Once a quadratic is written in this form, it can be solved using square roots.

2. Why Complete the Square?

$x^2 + 6x + 9 = 0$ cannot be solved by taking square root of both sides (the x variable is in two different terms)

But ***$(x + 3)^2 = 0$*** <u>can</u> be solved by taking square root of both sides since the ***variable is only in ONE term*** and easily isolated

Completing the square transforms a quadratic equation into this ***useful form***.

3. Steps for Completing the Square

(When the coefficient of x^2 is 1) Solve: $x^2 + 6x - 7 = 0$.

Isolate the x^2 and the x terms	$x^2 + 6x = 7$
Make space to complete square	$x^2 + 6x + __ = 7 + __$
Half the middle term coeff. and square	$x^2 + 6x + \underline{9} = 7 + \underline{9}$
Rewrite the perfect square	$(x + 3)^2 = 16$
Take the square root	$x + 3 = \pm 4$
Solve for variable	$x = 1$ or $x = -7$

Since we are isolating the x terms, we ADD the same value to both sides of the equation

*Taking the square root yields two values stemming from the **± nature of the square root.***

When a is NOT equal to 1, then follow the steps outlined in previous chapte for completing the square

5. Practice — Show All Steps

1. $x^2 + 8x + 8 = 0$
2. $x^2 - 4x + 12 = 0$
3. $2x^2 - 12x + 10 = 0$

6. Word Problem

A rectangular garden has a length 3 m greater than its width. Its area is 40 m². Let x be the width in metres.

a) Write a quadratic equation for the situation.

b) Solve by completing the square to find the width and length.

c) Explain why one mathematical solution must be rejected.

Homework:

PC 20 – 4.4 The Quadratic Formula and the Discriminant

DEVELOPING THE FORMULA BY COMPLETING THE SQUARE

Start with ***ax² + bx + c = 0***, where a ≠ 0. If you complete the square and solve for x in this format you can develop the Quadratic Formula.

ax² + bx + c = 0

THE QUADRATIC FORMULA

The values a, b, and c are the coefficients in a quadratic equation. The ***quadratic formula*** can solve any quadratic equation in this form.

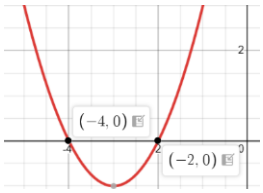
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**The symbol ± means that both the positive and negative possibilities must be considered.*

Let's do some simple examples.

Solve: $x^2+6x+8 = 0$

Solve by GRAPHING (4.1)



Solve by factoring (4.2)

Solve by Completing the Square (4.3)

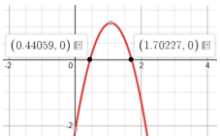
Solve by Using the Quadratic Formula (4.4)

THE DISCRIMINANT AND THE GRAPH

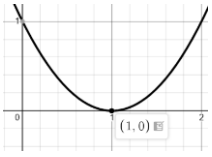
The discriminant (***D***) is ***$b^2 - 4ac$***, the expression under the square-root sign.

This value determines the NATURE of the ROOTS → 1. how many there are, and 2. whether they are real / non-real,

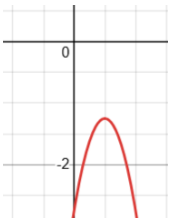
- If ***$b^2 - 4ac > 0$*** (pos), there are two distinct real roots; the parabola crosses the x-axis twice.



- If ***$b^2 - 4ac = 0$***, there is one distinct real root (two equal roots); the parabola touches the x-axis once.



- If ***$b^2 - 4ac < 0$*** (neg), there are no real roots; the parabola has no x-intercepts.



What is the nature of each solution for ***$-x^2+5x-7 = 0$***?

Homework: