PC 20 4.2 — Solving Quadratic Equations by Factoring

PART 1: Solving Quadratic Equations

1. What is a solution to a quadratic equation?

Recall: A quadratic equation is an equation that can be written in the form:

$ax^2 + bx + c = 0$, where $a \neq 0$

A solution is a value of x that makes the original equation true. The solutions of a quadratic equation are also called its ***roots***.

2. The zero-product principle

The zero-product principle states that if the ***product of two or more factors is zero, at least one of the factors must equal zero.***

For example:

$(x + 2)(x + 6) = 0$

In order for this to be true, at least one of the factors must equal zero. The two options come from:

$x + 2 = 0$ *or* $x + 6 = 0$

Solving gives us $x = -2$ *or* $x = -6$ these are the roots or solutions!

Important: The zero-product principle can only be applied when one side of the equation is zero. If an equation is not equal to zero, rearrange it first.

Example:

$x^2 + 8x = -12$	Move all terms to the left side:
$x^2 + 8x + 12 = 0$	Factor:
$(x + 2)(x + 6) = 0$	Apply the zero-product property:
$x + 2 = 0$ or $x + 6 = 0$	Solve

$x = -2, -6$

3. Factoring a trinomial

RECALL: A trinomial is a polynomial with **three** terms. For a quadratic trinomial of the form $x^2 + bx + c$, look for two numbers that:

- Multiply to give c.
- Add to give b.

Example:

$x^2 + 7x + 12 = 0$ The numbers 3 and 4 multiply to 12 and add to 7. Use these numbers in the binomial factors

$(x + 3)(x + 4) = 0$ Set each factor equal to zero:

$x + 3 = 0$ *or* $x + 4 = 0$ Therefore:

$x = -3, -4$

For quadratics with a leading coefficient other than 1, such as $2x^2 + 7x + 3$, look for factors that produce all three terms correctly. Use 'intuition' (mental math), or the 'decomposition' method (FM 10):

$2x^2 + 7x + 3 = 0$

$(2x + 1)(x + 3) = 0$

$x = -\frac{1}{2}, -3$

4. Solving by factoring

We have learned how to solve through *graphing* (identifying the *x-intercepts* of the graph).

Use the factors of a quadratic expression in this way:

1. Write the equation in standard form: $\boldsymbol{ax^2 + bx + c = 0}$.
2. ***Factor*** the quadratic expression completely.
3. Apply the ***zero-product principle*** by setting each factor equal to zero.
4. ***Solve*** each resulting equation.
5. ***State*** *solution(s)*
6. ***Check*** the solutions in the original equation when appropriate.

5. Factoring a difference of squares

Remember that you have learned how to factor different types of expressions. One of thoseA difference of squares is an expression in which one perfect square is subtracted from another.

The general pattern is:

$P^2 - Q^2 = (P - Q)(P + Q)$ Here, P and Q can be numbers, variables, or entire expressions.

Example 1:

$x^2 - 25 = 0$ Recognize that $25 = 5^2$.

$x^2 - 5^2 = 0$ Factor:

$(x - 5)(x + 5) = 0$ Set each factor equal to zero:

$x - 5 = 0$ or $x + 5 = 0$ Therefore: $\boldsymbol{x = 5, -5}$

Example 2:

$4x^2 - 49 = 0$ Recognize that $4x^2 = (2x)^2$ and $49 = 7^2$.

$(2x - 7)(2x + 7) = 0$ Therefore:

$2x - 7 = 0$ *or* $2x + 7 = 0$

$x = \frac{7}{2}, -\frac{7}{2}$

Remember: A difference of squares has a subtraction sign. An expression such as $x^2 + 25$ cannot be factored using this real-number pattern.

6. Factoring polynomials in quadratic form

Some expressions look more complicated than ordinary quadratics but **can be factored by temporarily replacing a repeated expression with a single variable.**

This strategy is called ***substitution***. It allows us to use familiar quadratic factoring methods.

Example:

Step 1: Let $w = x + 3$ $2(x + 3)^2 - 11(x + 3) + 15 = 0$

The equation becomes: $2w^2 - 11w + 15 = 0$

Step 2: Factor the quadratic $(2w - 5)(w - 3) = 0$

Step 3: Apply the zero-product property $2w - 5 = 0$ *or* $w - 3 = 0$

Solving for w: $w = \frac{5}{2}$ *or* $w = 3$

Step 4: Substitute back using w = x + 3 $x + 3 = \frac{5}{2}$ *or* $x + 3 = 3$

Solving for x: $x = -\frac{1}{2}$ *or* $x = 0$

Therefore, the solutions are: $x = -\frac{1}{2}, 0$

Important: Do not stop after solving for the temporary variable. Always substitute back to find the original variable.

7. Choosing the correct factoring method

When solving a quadratic equation, look for these patterns:

- **Greatest Common Factor**: Remove a factor shared by every term before doing anything else.
- ***Trinomial***: Look for two binomial factors that reproduce the original expression.
- **Difference of squares**: Recognize the pattern $P^2 - Q^2$.
- **Quadratic form**: Substitute a repeated expression with a temporary variable.

Not every quadratic equation factors easily using integers. If you cannot find suitable factors, another method, such as **completing the square** or the **quadratic formula**, may be required.

Final checklist

Before finalizing your work, ask yourself:

1. Is the equation equal to zero?
2. Have I factored completely?
3. Have I set every factor equal to zero?
4. Have I solved for the original variable?
5. Have I stated all solutions clearly?

Practice:

1. Solve: $(n + 3)(n - 1) = 0$

2. Solve: $4x^2 + 5x = 0$

3. Solve: $x^2 = -42 + 13x$

4. Solve: $15n^2 + 60n + 56 = -n$

5. Solve: $6(x+3)^2 - 12(x+3) - 18 = 0$

6. Solve: $4(k+6)^2 = 9(k+6)^2$

Homework: