4.1 Graphical Solutions to Quadratic Equations

A *quadratic* ***function*** *can be written in the form* ***y= ax² + bx + c*** as we've learned in Ch 3. This function consists of *many points* and *can be sketched as a graph.* A *quadratic* ***equation*** can be written in the form ***ax² + bx + c = 0*** (where a ≠ 0). One useful way to ***solve*** a quadratic equation is to ***find the solutions on the graph*** of its *corresponding* quadratic function.

For example, consider the quadratic *equation*: ***x² – 5x + 6 = 0***

The corresponding quadratic *function* is: ***y = x² – 5x + 6***

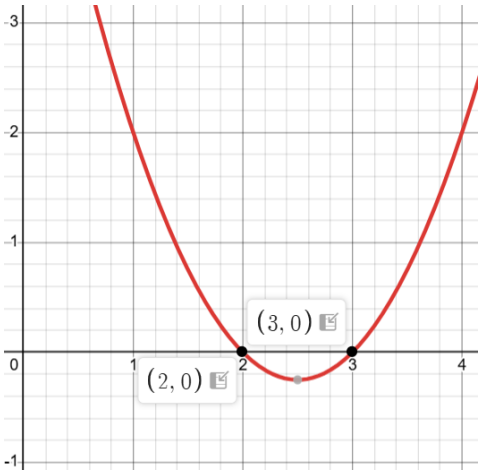
The points where the parabola crosses the x-axis ***(x-intercepts)*** are especially important because they ***provide the solutions*** to the original equation.

An ***x-intercept is a point on a graph*** where y = 0. Since the function has a value of zero at an x-intercept, the x-values of these points are also called the ***zeros*** of the ***function***.

For the example above, the graph ***y = x² – 5x + 6*** has x-intercepts at: ***(2, 0) and (3, 0)***

Therefore, the zeros of the function are: ***x = 2 and x = 3***

These values are also the ***roots / solutions*** of the quadratic ***equation***: ***x² – 5x + 6 = 0***



The solutions to the equation are ***x= 2, 3 NOT (2,0) and (3,0)***… the x-values of the intercepts, not the intercepts themselves!

This gives us an important connection:

Quadratic equation → corresponding quadratic function → graph → x-intercepts → zeros/roots → solutions

Why does this work?

The equation ***x² – 5x + 6 = 0*** can be viewed as asking:

"What value of x makes the expression x² – 5x + 6 equal to zero?"

The graph of ***f(x) = x² – 5x + 6* shows the value of the expression for different values of x. Whenever the graph reaches the x-axis, its y-value is 0. Therefore, the x-values at those locations solve the original equation.

Checking a Graphical Solution

It is always a good idea to check solutions by substituting them into the original equation.

For x = 2:
x² – 5x + 6 = 0
(2)² – 5(2) + 6 = 0
4 – 10 + 6 = 0
0 = 0
So x = 2 is correct.

For x = 3:
x² – 5x + 6 = 0
(3)² – 5(3) + 6 = 0
9 – 15 + 6 = 0
0 = 0
So x = 3 is also correct.

The **solutions of the quadratic equation** are therefore found by identifying the x values of the **x-intercepts**

*A ***solution*** should make the original equation ***true*** when substituted into it.

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The Number of Real Solutions

RECALL: A quadratic function is represented by a **parabola**. Depending on its position, the parabola can have zero, one, or two x-intercepts.

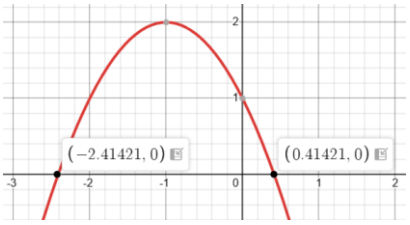
This means a ***quadratic equation can have zero, one, or two real roots***.

Case 1: Two Real Roots

If the ***parabola crosses the x-axis at two different points***, there are ***two x-intercepts***.

Therefore, the equation has:

- Two x-intercepts
- Two zeros
- Two distinct real roots
- Two real solutions

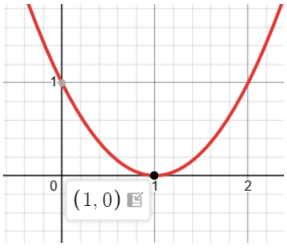


For example, if a graph crosses the x-axis at *(–2.41421,0) and (0.41241,0)* then ***x = –2.41421 and x = 0.41241*** are the **two solutions**.

Case 2: One Real Root

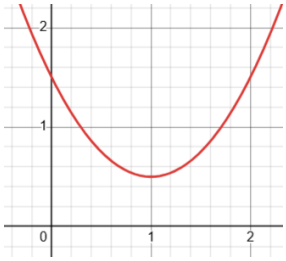
Sometimes the ***vertex*** of the parabola lies ***directly on the x-axis***. In this case, the parabola touches the x-axis at exactly one point, and there is only one x-intercept. The equation therefore has exactly **one real root**.

For example, if the graph touches the x-axis at x = 1, then x = 1 is the only real solution.

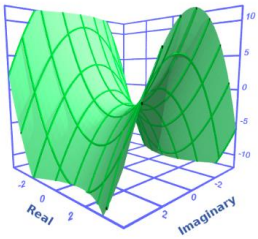


Case 3: No Real Roots

A parabola may also lie ***entirely above or entirely below the x-axis***. In this situation, the graph never reaches the x-axis. Therefore, there are no x-intercepts and ***no real solutions***.



It is important to remember that "no real roots" does not necessarily mean that the equation has no solutions of any kind. It means that there are no solutions that are **real numbers.*



Reading a Graph Carefully

When ***solving a quadratic equation graphically, focus on the x-axis*** rather than the y-axis.

Ask yourself:

- Does the parabola cross the x-axis?
- If so, where?
- Does it cross the x-axis twice, touch it once, or never reach it?
- What are the corresponding x-values of these 'touches'?

the ***x-intercept is a zero of the graph***, and the ***solution is the root of the equation***

Remember that the ***x-coordinate*** of an ***x-intercept*** *gives the* ***solution***. The y-coordinate will always be 0 and is NOT part of the solution

Homework: