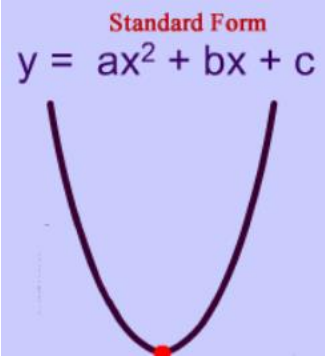
3.2 Investigating Quadratic Functions in Standard Form

1. Understanding Quadratic Functions

A quadratic function is a function whose highest exponent on the variable is 2.

Recall the Vertex Form of a Quadratic equation is $f(x) = a\,(x - p)^2 + q$

The ***Standard Form*** of a quadratic function is: $\boldsymbol{f(x) = ax^2 + bx + c}$ where $\boldsymbol{a \neq 0}$



The letters **a, b, and c** are coefficients that influence the ***location and shape*** of the graph. The *value of* ***'a'*** determines the ***width and direction the graph opens.*** *This is the same 'a' as in the VERTEX form.*

The values of ***b and c are not in the vertex form***, but *can be used to find the axis of symmetry and the y-intercept.*

Recall the graph of every quadratic function is a smooth, U-shaped curve called a parabola. A parabola can open upward or downward, depending on the value of 'a'.

2. Important Features of a Parabola

For a quadratic function in standard form, the ***equation of the axis of symmetry*** is: $\boldsymbol{x = \frac{-b}{2a}}$

This means $\boldsymbol{x = \frac{-b}{2a}}$ also determines the ***x-value of the vertex (***and thus means that $\boldsymbol{p = \frac{-b}{2a}}$***)***

The ***y-intercept*** is the point where the graph crosses the y-axis and at this point, x = 0.

Substituting $x = 0$ $\boldsymbol{f(0) = a(0)^2 + b(0) + c}$ and thus gives $f(0) = c$

For a quadratic function in standard form, the ***y-intercept is (0, c)***

The ***x-intercepts*** are the points where the graph crosses or touches the x-axis. ***At these points, y = 0, so they are also called the zeros or roots of the function.*** *A parabola may have two, one, or no x-intercepts.* The quadratic expression will need to be factored to determine x-intercepts. Example to follow later.

3. Finding the Vertex and Other Features

To investigate a quadratic function in standard form, we can use its equation to calculate important features without having to graph every point.

Example: $\boldsymbol{f(x) = x^2 - 6x + 5}$

Step 1: Identify *a, b,* and *c.*
a = 1, b = −6, c = 5

Step 2: Find the *axis of symmetry*.
$\boldsymbol{x = \frac{-b}{2a}} = \frac{-(-6)}{2(1)} = \frac{6}{2} = 3$
The axis of symmetry is $\boldsymbol{x = 3}$.

Step 3: Find the vertex.
The x-coordinate ($\boldsymbol{x = \frac{-b}{2a}}$) is ***3***. Substitute x = 3 into the original function:
$\boldsymbol{f(3) = 3^2 - 6(3) + 5} = 9 - 18 + 5 = \boldsymbol{-4}$
The vertex is *(3, −4).* Since *a = 1* is positive, the parabola opens upward. The vertex is the minimum point, and the minimum value is *−4*.

Step 4: Find the y-intercept.
The y-intercept is *(0, c)*. Since c = 5, the ***y-intercept is (0, 5)***

Step 5: Find the x-intercepts.
Set the function equal to zero and solve:
$x^2 - 6x + 5 = 0$
Factor the trinomial
$(x - 1)(x - 5) = 0$
Therefore, x = 1 or x = 5. The ***x-intercepts are (1, 0) and (5, 0)***

Your turn:
Find the important features of ***$f(x) = 2x^2 - 4x - 6$*** without having to graph every point (as in the example above).

4. Domain, Range, and Applied Situations

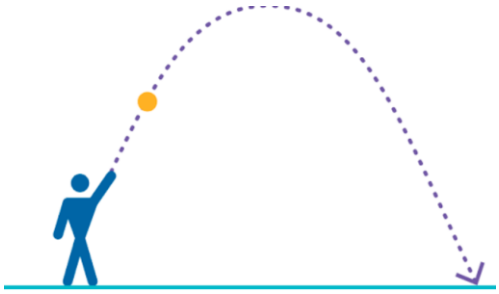
The ***domain is the set of all possible x-values*** of a function. For a quadratic function the domain is all real numbers. ***$D = \{x \in R\}$***

The ***range is the set of all possible y-values*** of a function. It ***depends on the vertex and the direction*** in which the parabola opens.

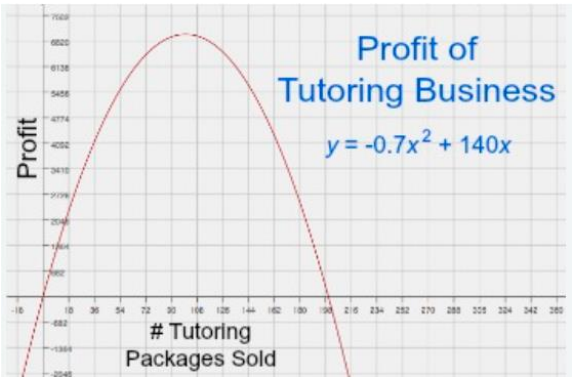
• For a parabola opening **upward**, the range includes all ***y-values greater than or equal to the y-coordinate of the vertex***.
• For a parabola opening **downward**, the range includes all ***y-values less than or equal to the y-coordinate of the vertex***.

For ***$f(x) = x^2 - 6x + 5$*** *in the example above*, the ***domain is all real numbers*** *and the* ***range is $y \geq -4$***.

In applied situations, quadratic functions can model real-world relationships such as the *path of a ball*, the *height of a projectile*, or the *profit earned by a business*.



The intercepts and vertex have practical meanings. For example, the vertex might represent the maximum height of a ball, while an x-intercept could represent the time when the ball reaches the ground. The *domain and range may need to be restricted* to values that make sense in the situation. A ball's flight, for instance, does not continue for negative time, or after it has landed.



Understanding these features allows us to interpret a quadratic equation, sketch its graph, and explain what its mathematical results mean in a real-world context.

Q: For the Profit graph shown here, HOW MANY PACKAGES should this company plan to make in order to MAXIMIZE their profits? What would that expected profit be?

Now, do these values match with the equation? Prove.

Homework:

https://www.mrmathwell.com/index.php/courses/pre-calculus-20/