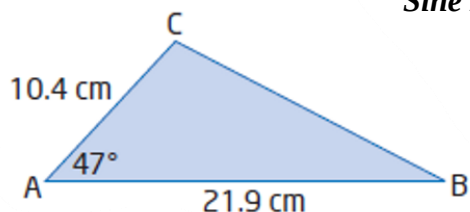


PRECALCULUS 20 – SECTION 2.4 - THE COSINE LAW

Cosine Law

The **Cosine Law** is a trigonometric relationship that can be used to solve triangles that are *not right triangles (SOH CAH TOA)*, and *where the Sine Law cannot be used (no angle opposite side pairing given)*.

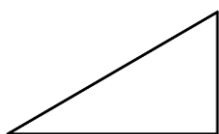


The cosine law can be used in two main situations. First, it can be used to find an unknown side when two sides and the **included angle** are known (shown).

Second, it can be used to **find an unknown angle when all three sides are known**.

A Useful Connection to the Pythagorean Theorem

The Cosine Law $a^2 = b^2 + c^2 - 2bc \cos A$ is closely related to the **Pythagorean Theorem**. The cosine law includes a small adjustment to the Pythagorean theorem ($- 2bc \cos A$), which accounts for the slightly longer lengths of a and b.



Consider what happens when the included angle is 90° :

$$a^2 = b^2 + c^2 - 2bc \cos 90^\circ$$

Since $\cos 90^\circ = 0$, then $2bc \cos 90^\circ = 0$

$$a^2 = b^2 + c^2$$

This is exactly the Pythagorean theorem.

The **Cosine Law** therefore extends the Pythagorean theorem so that it can be used with triangles having *acute, right, or obtuse angles*.

Finding an Unknown Side

Suppose you know sides b and c and the angle between them, angle A. The side opposite angle A is side a. The Cosine Law states:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Similarly:

$$b^2 = a^2 + c^2 - 2ac \cos B \quad \text{and} \quad c^2 = a^2 + b^2 - 2ab \cos C$$

The most important idea is to **match the angle with the side directly opposite it**. For example, if you know angle B and want to find side b, use:



Solving a Triangle Using the Cosine Law

When solving a problem, first identify what information is known and what is unknown. Draw or label the triangle if necessary. Determine which version of the Cosine Law matches the angle and side you are working with.

For example, suppose:

$$b = 8, c = 11, \text{ and } A = 60^\circ$$

To find a:

$$a^2 = 8^2 + 11^2 - 2(8)(11)\cos 60^\circ$$

$$a^2 = 64 + 121 - 176(0.5)$$

$$a^2 = 97$$

$$a = 9.85$$

Therefore, the unknown side is approximately 9.85 units long.

Keep your calculator in the correct **degree mode when the angles are given in degrees.*

Finding an Unknown Angle

The Cosine Law can also be rearranged to find an unknown angle when all three side lengths of a triangle are known.

Starting with: $a^2 = b^2 + c^2 - 2bc \cos A$

$$\cos A = \frac{a^2 - b^2 - c^2}{-2bc}$$

Example: Finding an Angle

Suppose a triangle has:

$$a = 7, b = 8, c = 10$$

find angle C:

Angle C is approximately 83.3°

Cosine Law and Sine Law Together

The **Sine Law and Cosine Law** can be used together when solving a triangle. The Cosine Law can be particularly useful for finding a missing side or angle when the information does not initially provide a convenient Sine Law relationship.



When solving a complete triangle, also remember that the three angles must add to 180° :

$$\angle A + \angle B + \angle C = 180^\circ$$

Choosing the Correct Law

A useful strategy is to look at the information you have before choosing a formula.

If you know **two sides and the included angle**, the Cosine Law can be used to find the third side.

If you know **three sides**, the Cosine Law can be rearranged to find an angle.

If you already know an angle and its opposite side, along with one other side or angle, the Sine Law may be more convenient.

The Cosine Law and Sine Law are therefore an important tool for solving **oblique triangles**, meaning triangles that are not necessarily right triangles.

Common Errors to Avoid

1. Make sure the **angle is matched with the side opposite** it.
2. Use the **two sides that form the included angle in the final term**.
3. Use **parentheses** when entering the formula into a calculator.
4. Check that your calculator is in **degree mode** when working with degree measures.
5. When finding an angle, use **inverse cosine, written $\cos^{-1}$** , rather than ordinary cosine.
6. **Round only at the end** of the calculation unless instructed otherwise.
7. Check whether your **answer is reasonable** for the triangle.

Homework:

