

PC 20 2.2 – TRIGONOMETRIC RATIOS OF ANY ANGLE

Learning Goal: Use the coordinates of a point on the terminal arm of an angle in standard position to determine trigonometric ratios, determine their signs from the quadrant, and recognize quadrantal angles.

1. Trigonometric Ratios Using Coordinates

When an angle θ is in standard position, its initial arm lies on the positive x-axis and its vertex is at the origin. If a point $P(x, y)$ lies on the terminal arm, we can use its coordinates to find the three primary trigonometric ratios.

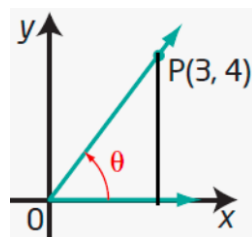
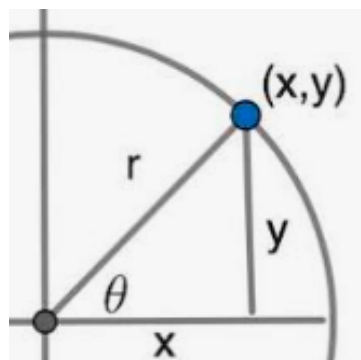
First calculate the distance from the origin to P . This distance is called r and is found using the Pythagorean theorem: $r = \sqrt{x^2 + y^2}$. The value of r is always positive because it represents a distance.

Based on SOH CAH TOA, the three primary ratios are now

$$\sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r} \quad \tan \theta = \frac{y}{x}$$

S Y R C X R T Y X

These formulas work for angles in any quadrant, provided the coordinates of a point on the terminal arm are known.



Eg. if $P(3, 4)$ is on the terminal arm, then $x=3$, $y=4$, and

$$r = \sqrt{3^2 + 4^2} =$$

Therefore $\sin \theta = \frac{4}{5} \quad \cos \theta = \frac{3}{5} \quad \tan \theta = \frac{4}{3}$

2. Signs of Trigonometric Ratios

The **signs of x and y** determine whether each trigonometric ratio is positive or negative.

C.A.S.T. Rule

An important way to remember *signs of trig functions* in quadrants is the **CAST Rule**. Starting in quadrant IV and working counterclockwise, the letters help remember which trig function is positive in which quadrant.

In **Quadrant I**, both x and y are positive, so **all** three primary ratios are positive.

In **Quadrant II**, x is negative and y is positive. Therefore, **sine** is positive, while cosine and tangent are negative.

In **Quadrant III**, **tangent** is positive.

In **Quadrant IV**, **cosine** is positive.

S	A
$\cos \theta = \text{negative}$ $\sin \theta = \text{positive}$ $\tan \theta = \text{negative}$	$\cos \theta = \text{positive}$ $\sin \theta = \text{positive}$ $\tan \theta = \text{positive}$
T	C
$\cos \theta = \text{negative}$ $\sin \theta = \text{negative}$ $\tan \theta = \text{positive}$	$\cos \theta = \text{positive}$ $\sin \theta = \text{negative}$ $\tan \theta = \text{negative}$



3. Determining a Ratio from a Point

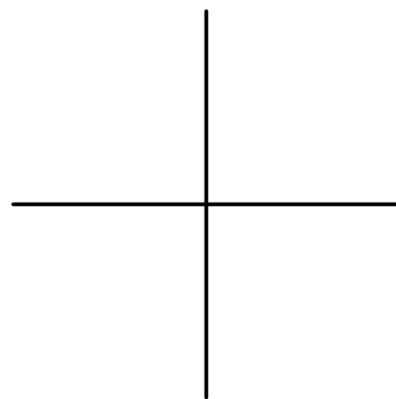
Suppose $P(-5, 12)$ lies on the terminal arm of θ . The point is in Quadrant II because x is negative and y is positive.

Calculate r first: $r = \sqrt{(-5)^2 + 12^2} = \sqrt{169} = 13$

Now substitute into the coordinate formulas:

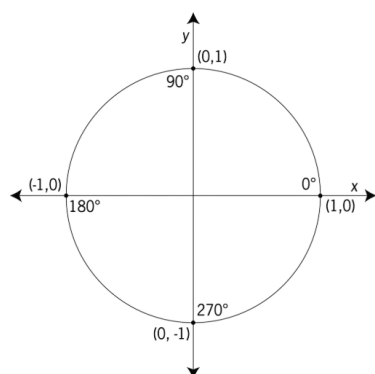
$$\sin \theta = \frac{12}{13} \quad \cos \theta = \frac{-5}{13} \quad \tan \theta = \frac{12}{-5}$$

Notice that the signs agree with the Quadrant II sign pattern.



4. Quadrantal Angles

A **quadrantal angle** is an angle in standard position whose terminal arm lies directly on one of the coordinate axes. The common quadrantal angles from 0° to 360° are 0° , 90° , 180° , 270° , and 360° .



For a quadrantal angle, **one coordinate may be zero**. This means some trigonometric ratios may be **zero**, or (for tangent only) **undefined** because division by zero is not possible.

$$\sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r} \quad \tan \theta = \frac{y}{x}$$

5. Exact Values vs. Approximate Decimal Values

An **exact value** is a value written without rounding,

such as $\frac{3}{5}$, $-\frac{\sqrt{2}}{2}$ or $\frac{3}{5}$. An approximate decimal value is a rounded numerical form, such as 0.62 or -0.707. Unless a question specifically asks for a decimal approximation, an **exact value is usually preferred**.

When a decimal is required, keep lots of digits during your calculation and round only at the end. The wording of a question should tell you whether an exact value or an approximate decimal value is expected. Rounding for *trig function values* is usually 4 decimal places, all other cases try to keep 3 decimal places in your answer.

Homework:

