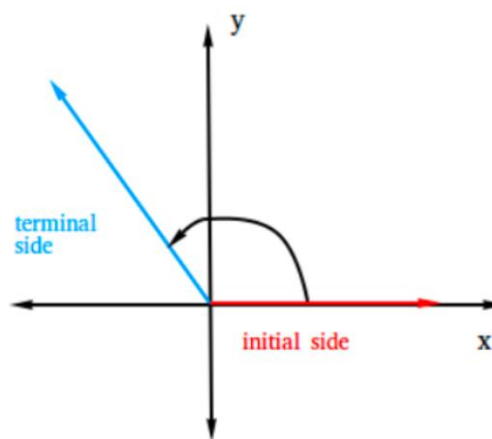


PRECALCULUS 20 2.1 – ANGLES IN STANDARD POSITION

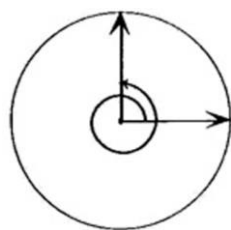
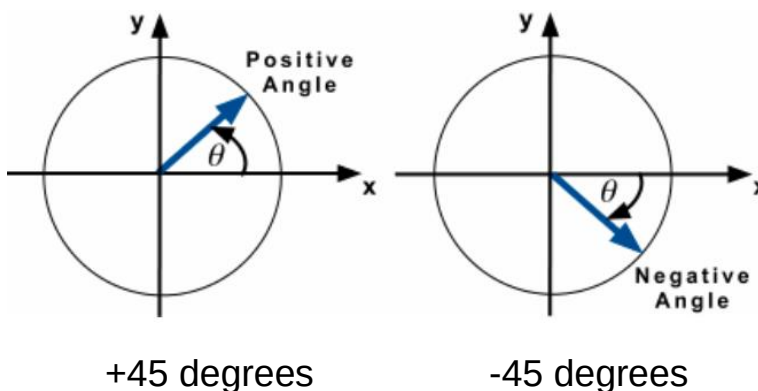
Angles in Standard Position

An **angle** is formed when one ray rotates about a common endpoint, called the vertex. An **angle** in **standard position** has its vertex at the **origin**, $(0, 0)$, and its **initial arm** lies along the positive x-axis. The **initial arm** is the side where the rotation begins, while the **terminal arm** is the side where the rotation finishes.



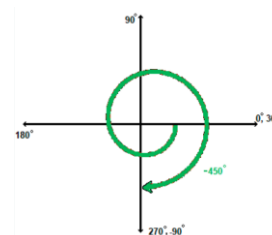
The direction of rotation determines whether an **angle** is positive or negative. A **counterclockwise** rotation is positive, while a **clockwise** rotation is negative.

For example, 45° is a positive **angle** because it rotates **counterclockwise** from the positive x-axis. An **angle** of -45° rotates **clockwise** from the same starting position.



Angles can be greater than one complete rotation. One complete rotation is 360° . Therefore, 450° makes one full rotation and then continues another 90° . Likewise, -450° represents a **clockwise** rotation of one full turn followed by another 90° **clockwise**.

When an angle's terminal arm lies on the x or y axis, it is called a **quadrantal angle**.



Quadrants

The coordinate plane is divided into four **quadrants**. **Quadrant I** has positive x and positive y coordinates. **Quadrant II** has negative x and positive y coordinates. **Quadrant III** has negative x and negative y coordinates. **Quadrant IV** has positive x and negative y coordinates.

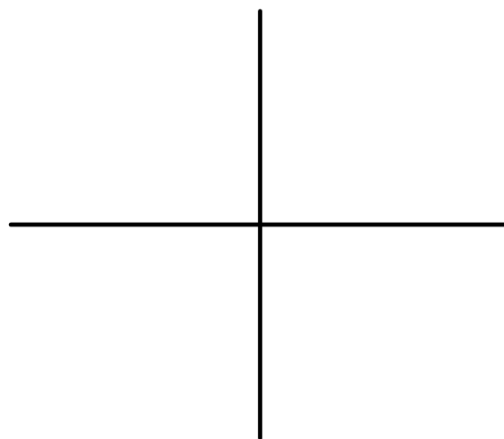
The quadrant of an angle is determined by the location of its **terminal arm**.

For example, 135° lies in **Quadrant II**,

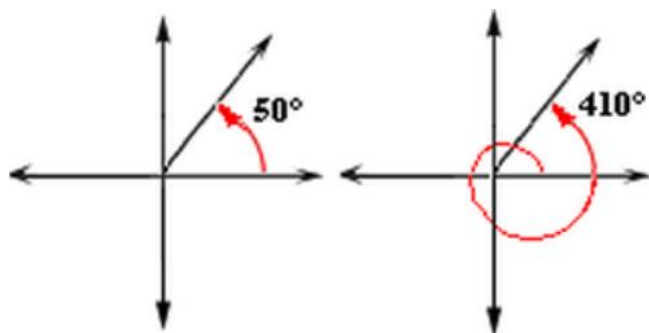
225° lies in **Quadrant III**, and

315° lies in **Quadrant IV**.

A negative angle is measured clockwise, so -45° has its **terminal arm** in **Quadrant IV**.



Coterminal Angles



Coterminal angles are angles that have the same **terminal arm** in standard position. They differ by one or more complete rotations. For example, 45° , 405° , and -315° are coterminal because adding or subtracting **360°** does not change the **terminal arm**.

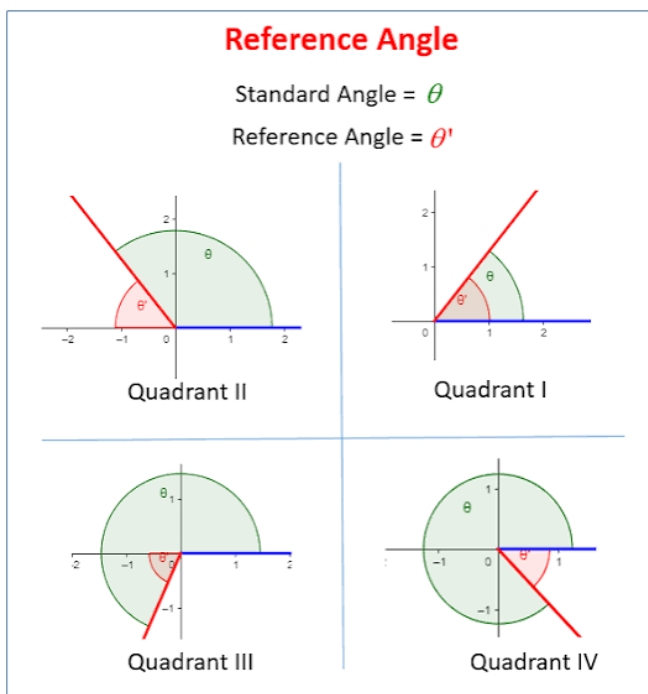
A useful rule for finding **coterminal angles** is $\theta + 360^\circ n$, where n is any integer. Positive n means add 360° and negative n means minus 360° . This means that an angle has infinitely many **coterminal angles**.



Reference Angles

A **reference angle** is the positive, **acute** angle between the **terminal arm** and the **x-axis**. It is useful because it connects a larger or less familiar angle to an **acute** angle such as 30° , 45° , or 60° . A **reference angle** is between 0° and 90° when the **terminal arm** is not on an axis.

The rule used to find a **reference angle** depends on the quadrant. In **Quadrant I**, the **reference angle** equals θ . In **Quadrant II**, use $180^\circ - \theta$. In **Quadrant III**, use $\theta - 180^\circ$. In **Quadrant IV**, use $360^\circ - \theta$.



For example, 150° is in **Quadrant II**, so its **reference angle** is $180^\circ - 150^\circ = 30^\circ$. The angle 225° is in **Quadrant III**, so its **reference angle** is $225^\circ - 180^\circ = 45^\circ$. The angle 315° is in **Quadrant IV**, so its **reference angle** is $360^\circ - 315^\circ = 45^\circ$.

Special Triangles and Exact Values

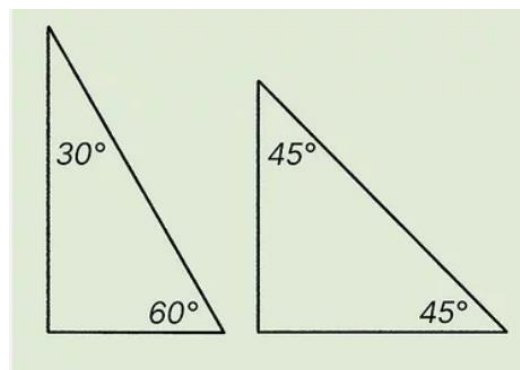
The angles 30° , 45° , and 60° occur frequently in trigonometry. Their exact trigonometric ratios can be found using **special triangles**. A **30° - 60° - 90° triangle** and a **45° - 45° - 90° triangle**.

An **exact value** is written without rounding. For

example, $\sin 60^\circ = \frac{\sqrt{3}}{2}$ is an **exact value**.

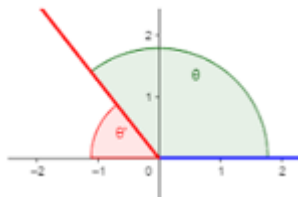
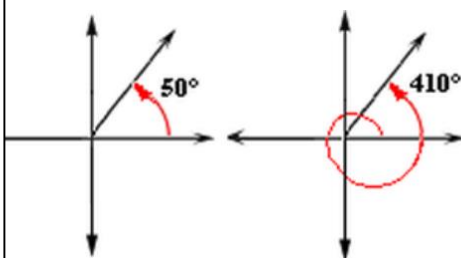
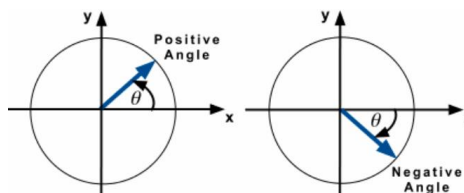
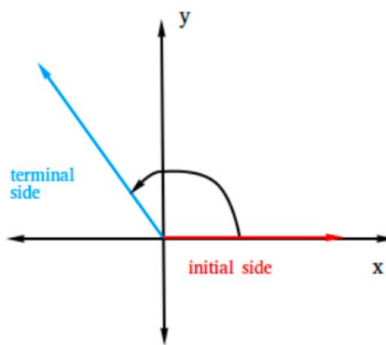
Recall: SOH, CAH, TOA

A calculator may give $\sin 60^\circ \approx 0.8660$, which is an **approximate decimal value**. The symbol $\approx$ means approximately equal to. A decimal result has normally been rounded, so it is an approximation rather than the **exact value**.

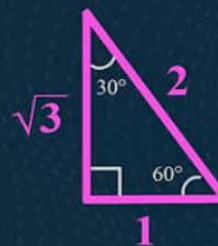
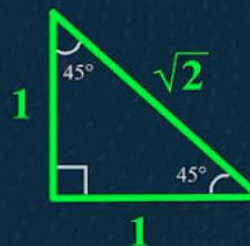


Key Ideas

- Standard position: vertex at the origin and initial arm on the positive x-axis.
- The terminal arm shows where the angle finishes.
- Counterclockwise rotations are positive; clockwise rotations are negative.
- Coterminal angles have the same terminal arm and differ by multiples of 360° .
- A reference angle is the positive acute angle between the terminal arm and the x-axis.
- Special triangles provide exact trigonometric values for 30° , 45° , and 60° .
- An exact value is unrounded; a calculator decimal is an approximate value.



$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

Special Right Triangles

Homework:

