

PRE-CALCULUS 20 Geometric Sequences and Series — P20.10**Learning Outcome**

P20.10 — Demonstrate understanding of geometric (finite and infinite) sequences and series.

This package separates the topic into **Section 1.3: Geometric Sequences**, **Section 1.4: Geometric Series**, and **Section 1.5 infinite Geometric Series**

PC 20 1.3 GEOMETRIC SEQUENCES**What Is a Geometric Sequence?**

A **sequence** is an ordered list of numbers that follows a pattern. A **geometric sequence** is a sequence in which each term is **multiplied** by the same number to obtain the next term. This number is called the **common ratio**

Example: 2, 6, 18, 54, 162, ...

Each term is multiplied by 3, so the **common ratio** is 3.

Finding the Common Ratio

Divide any term by the term immediately before it: $r = t_2 / t_1$. The ratio must remain constant for the sequence to be geometric.

3, 6, 12, 24, ... is geometric because

$$6/3 = 12/6 = 24/12 \quad \text{and } r = 2.$$

The sequence 5, 10, 15, 20, ... is **not geometric** because it has a **constant difference rather than a constant ratio**. (Recall this is arithmetic)

The General Term

The **general term** formula is:

$$t_n = t_1 r^{n-1}$$

t_n is the actual term in the n th position of the geometric sequence

t_1 is the first term,

r is the common ratio

n is the position of the term

Example

Find the 6th term of 3, 6, 12, 24, ...

$$t_1 = 3, r = 2, n = 6$$

$$t_6 = 3(2)^5 = 96$$

Therefore, the sixth term is 96.

Visually $\xrightarrow{\quad} \xrightarrow{\quad} \xrightarrow{\quad} \xrightarrow{\quad} \xrightarrow{\quad}$
3, 6, 12, 24, __, __

Why $n - 1$?

Visually, we see that we 'jump' $n-1$ times to get from term 1 to term n .

Also, the first term has been multiplied by the ratio zero times: $t_1 = t_1 r^0$. The second term uses r^1 , the third uses r^2 , and so on. Therefore, the exponent is $n - 1$.



Using the Formula to Find Missing Information

The formula $t_n = t_1 r^{n-1}$ can be rearranged or solved to find a missing first term, ratio, or term position.

Example: Find the First Term

If $t_7 = 192$ and $r = 2$, find t_1 :

Your Turn: If $t_6 = 972$ and $r = 3$, find t_1 :

$$192 = t_1 (2)^6$$

$$192 = 64t_1$$

$$\text{so } t_1 = 3.$$

Example: Find the Position

If $t_1 = 2$, $r = 3$, and a term is 486, find n : Your Turn: If $t_1 = 5$, $r = -2$, and a term is -2560, find n :

$$486 = 2(3)^{n-1}$$

$$243 = 3^5,$$

$$n - 1 = 5$$

$$n = 6$$

Geometric Sequences in Real Situations

Geometric sequences are useful when a quantity changes by the same **factor** repeatedly. Examples include population growth, depreciation, repeated percentage changes, investment growth, and other situations involving multiplication.

Repeated Percentage Change

If an amount **increases by 10%** each period, the **multiplier is 1.10**. If it **decreases by 15%**, the **multiplier is 0.85**. The multiplier—not the percentage itself—is the common ratio.

Key Strategy

To work with a geometric sequence: identify t_1 , determine r , identify n , and substitute into $t_n = t_1 r^{n-1}$. Always check that the ratio is constant.

Homework:



PC 20 1.4 GEOMETRIC SERIES**From Sequences to Series**

A **series** is formed by adding the terms of a sequence.

Sequence: 2, 6, 18, 54, ...

Series: $2 + 6 + 18 + 54 + \dots$

A **finite series** has a specific number of terms. An **infinite series** continues without a final term.

Sum of a Finite Geometric Series

For a **finite** geometric series, the sum CAN be obtained. The sum of the first n terms is:

$$S_n = \frac{t_1(1 - r^n)}{(1 - r)}$$

NOTE: this is the same formula $S_n = \frac{t_1(r^n - 1)}{(r - 1)}$

where S_n is the sum, t_1 is the first term, r is the common ratio, and n is the number of terms.

Example

Find the sum of $3 + 6 + 12 + 24 + 48$.

Your Turn: Find the sum of $-5 + 10 + (-20) + 40$.

$$t_1 = 3, r = 2, n = 5$$

$$S_5 = \frac{3(1 - 2^5)}{(1 - 2)}$$

Situational Example: Bouncing Ball

A ball rebounds to 60% of its previous height. If its first rebound is 6 m, the rebound heights are 6, 3.6, 2.16, 1.296, ... This is geometric with $r = 0.60$.

The first four rebound heights total 13.056 m. A finite-series formula can find this total efficiently without adding every term one at a time.

When the **final term** in a geometric series (t_n) is **KNOWN** you may use this formula:

$$S_n = \frac{t_1 - rt_n}{(1 - r)}$$

Eg. find the sum of the first 5 terms of the series: 3, 9, ...243

Important Distinction

A sequence lists terms; a series adds terms. A finite series has a set number of terms, while an *infinite series continues indefinitely*.

Homework:



PC 20 1.5 INFINITE GEOMETRIC SERIES**Infinite Geometric Series**

An infinite geometric series **can have a finite sum** when the terms become smaller and approach zero. That is **when $|r| < 1$** .

For t_1 = first term and common ratio r :

$$S_{\infty} = \frac{t_1}{(1-r)} \quad \text{*This formula applies only when } |r| < 1.$$

Convergence and Divergence

An infinite series is **convergent** if its partial sums approach a finite value.

This means a series is convergent when the list of numbers added gets closer to one value

Eg. $0.1 + 0.01 + 0.001 + 0.001 + \dots$ these numbers are getting closer to zero

It is **divergent** if it does not approach a finite value but gets larger or smaller without converging to a number

Eg. $2 + 3 + 9/2 + 27/4 + \dots$

Negative Ratios

A negative ratio does not automatically cause divergence. For example, $8 - 4 + 2 - 1 + \dots$ has $r = -1/2$. Because $|-1/2| < 1$, it converges.

Your Turn: What is the ratio here and does it converge? $2-4+8-16+\dots$

The test is simple: if $|r| < 1$, the infinite geometric series converges. If $|r| \geq 1$, it diverges.

Example: Convergent

$4 + 2 + 1 + 0.5 + \dots$ has $r = 1/2$. Since $|1/2| < 1$, it converges and has a sum.

$$S_{\infty} = \frac{t_1}{(1-r)}$$

$$S_{\infty} = \frac{4}{(1-\frac{1}{2})}$$

Example: Divergent

$3 + 6 + 12 + 24 + \dots$ has $r = 2$. Since $|2| > 1$, it diverges and **has no finite sum**.

Common Mistakes

Do not confuse a sequence with a series. Do not use the infinite-sum formula when $|r| \geq 1$. Do not assume a negative ratio means divergence.

Homework:

