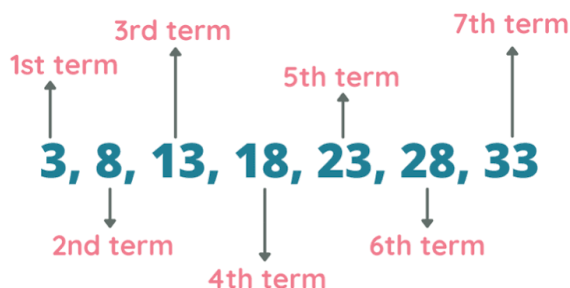


PRECALCULUS 20 Section 1.1 — Arithmetic Sequences

A **sequence** is an ordered list of *elements* that follows a rule or pattern. Each element, usually a number, is called a term. The position of a term is represented by n . Thus, t_1 is the first term, t_2 is the second term, and t_n is the term at position n . The first term is also commonly represented by a .



An **arithmetic sequence** is a sequence in which the *difference* between consecutive terms is *constant*. This constant difference is called the **common difference, d** .

A sequence can have an identifiable pattern without being arithmetic. For example, 2, 4, 8, 16, 32 follows a pattern because each term is multiplied by 2, but it is not arithmetic because its consecutive differences are not constant.

Eg. 2, 4, 8, 16, 32

Example: 3, 8, 13, 18, 23, 28, 33, ...

$d = t_n - t_{n-1}$ or any term, minus the term before it in the sequence

$d = 8 - 3 = 5$, (every following difference is also 5) so the **common difference is 5**

Visual — equal steps on a number line:

3 —(+5)— 8 —(+5)— 13 —(+5)— 18 —(+5)— 23...

The assumption that makes a sequence arithmetic is that the **same difference** continues from one term to the next. ***The common difference may be positive, negative, or zero.***



Generating an arithmetic sequence

If $t_1 = 4$ and $d = 6$:

$4 \rightarrow 10 \rightarrow 16 \rightarrow 22 \rightarrow 28 \rightarrow 34 \rightarrow \dots$

or **4,10,16,22,28,34**

Your turn: $t_1 = -5$ and $d = -4$

Nth-term formula (General Term Formula)

$$t_n = t_1 + (n - 1)d$$

This formula finds a term without listing every term.

For example, if $t_1 = 7$ and $d = 4$, then

$$t_{10} = 7 + (10 - 1)(4) = 43.$$

Key variables

t_n = nth term

t_1 = first term

n = position of term

d = common difference

To solve for missing information, substitute what is known into the nth-term formula and rearrange when necessary.

For example: Solve for **d** and for **n**

Solve for d: $t_1 = 5$, $t_8 = 26$

Solve for n given: $t_1 = 9$, $t_n = -18$, $d = -3$

Homework:



PRECALCULUS 20 Section 1.2 — Arithmetic Series

A **series** is formed when the terms of a sequence are *added up*.

For example, the sequence 5, 8, 11, 14, 17 becomes the series $5 + 8 + 11 + 14 + 17$.

The symbol S_n represents the ***sum of the first n terms***.

When the final term is known

$$S_n = \frac{n(t_1 + t_n)}{2}$$

Use this when you know the number of terms, the first term, and the final term.

When the final term is unknown - substitute the t_n formula into this formula

For the sum of the 5 terms in this sequence (S_5) notice the pattern that gives the formula...

$$5 + 8 + 11 + 14 + 17$$

$$+ \quad 17 + 14 + 11 + 8 + 5$$

$$(22 + 22 + 22 + 22 + 22)$$

Divide by two and you have the sum.

$S_n = \frac{n[2t_1 + (n-1)d]}{2}$ Use this when you know the number of terms, the first term, and the common difference.

Real-life applications

Arithmetic sequences can model situations such as **savings**, wages, seating arrangements, construction patterns, production, and distances whenever the amount changes by a constant amount.



Practice:

1. How many gifts were given to me by my 'true love' on the 12th day of Christmas???
2. Find t_5 and S_5 given: $10 + 7 + 4 + \dots$
3. Determine the sum of the following series: $5 + 8 + 11 + \dots + 53$ (careful this is a two stepper!)
4. The sum of the first 5 terms of an arithmetic series is 65, the sum of the first 6 terms is 87. What are the first three terms of the sequence?

<u>Homework:</u>

