

6.4 / 6.5 Optimization Problems

What Is an Optimization Problem?

An **optimization problem** is a situation where you are trying to find the **best possible outcome**. This usually means finding a **maximum** (largest value) or a **minimum** (smallest value).

Examples include:

- Maximizing profit
- Maximizing production
- Minimizing cost

Important Terms

- **Optimization problem:** A problem where a quantity must be **maximized** or **minimized**
- **Variables:** Quantities that can change (e.g., number of cars, number of products)
- **Constraints:** Limits or conditions in the problem
- **Linear Inequalities:** Mathematical statements that describe constraints

New for this section

- **Objective Function:** An *equation* representing the relationship between the variables and the quantity to be maximized or minimized.

**The Initial Steps Involved in Optimization are the same as graphing linear Inequalities...*

You might:

- **Minimize cost**, or
- **Maximize revenue**

Step 1: Define the Variables

Start by identifying what can change.

Example:

Let

c = number of cars

m = number of minivans

Step

2: Identify Restrictions

Restrictions tell you what values the variables are allowed to take.

Examples:

- You cannot produce negative items $\rightarrow c \geq 0, m \geq 0$
- Only whole items $\rightarrow c, m$ are whole numbers

These are called **restrictions**.

Step 3: Write Constraints

Constraints come directly from the problem and are written as **linear inequalities**.

Example:

- "No more than 12 cars" $\rightarrow c \leq 12$
- "At least 70 vehicles total" $\rightarrow c + m \geq 70$

Each condition becomes a constraint.

NEW! Step 4: Write the Objective Function (EQUATION)

The **objective function** represents *how the variables are related to the quantity to be optimized*.

Example:

- If the goal is to MINIMIZE COST, and the cost is \$8 per car and \$12 per SUV:

$$\text{Minimized Cost is } Q = 8c + 12m$$

Step 5: Graph the System

All **constraints are graphed** on a coordinate plane.

(Do NOT graph the Objective Quantity as part of the system)

The overlapping region is called the **feasible region**

This region contains all possible solutions that satisfy the constraints.

Recall: Feasible Region is where all constraints overlap. Only points inside this region are valid solutions

What Happens Next?

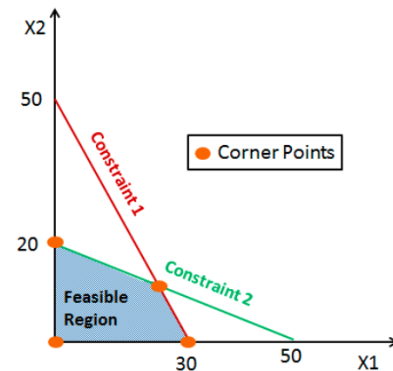
Each point in the feasible region represents a possible solution.

To find the **optimal solution for the objective function**, you:

- Substitute combinations of variable values into the objective function
- **Compare** the results

**Important Observations

- Maximum and minimum values (Optimal Solutions) usually occur at the **corner points** of the feasible region
 - For any situation, the **extremes** of all combined constraints will occur at the corners of the feasible region.
 - By contrast, points in the **middle of the region** do not maximize any of the constraints... there is always “a little more possible” for each constraint

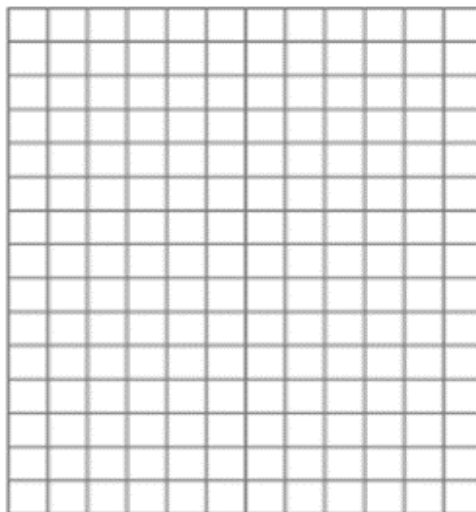


Example: Maximizing Revenue (Florist Problem)

Scenario: A florist wants to maximize revenue by selling two types of bouquets: **Standard** and **Deluxe**.

- **Constraints:**
 - **Time:** A Standard bouquet takes 10 minutes to make, and a Deluxe takes 20 minutes. The florist has a maximum of 400 minutes available.
 - **Flowers:** The florist has a maximum of 300 flowers available. Standard bouquets use 10 flowers, and Deluxe bouquets use 20.
 - **Demand:** Due to space, the total number of bouquets cannot exceed 25.
- **Goal:** Maximize revenue, where Standard bouquets sell for \$30 and Deluxe for \$50

Solution:



Summary Steps

To create a model:

1. Identify **OBJECTIVE QUANTITY** to **maximize** or **minimize**
2. Define your **variables**
3. Identify **restrictions**
4. Write **constraints (linear inequalities)**
5. Write the **objective function**
6. Graph and identify the **feasible region**
7. **Test vertices** (corner points) to find maximum or minimum values for the Objective Quantity

Textbook Questions: