

### Apply Your Understanding of Summation Notation

In the chart below, the limit of a Riemann sum has been provided for you. Write the corresponding definite integral.

Definite Integral	Limit of Riemann Sum
1.	$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left[ \sqrt{2\left(\frac{6}{n}\right) + 1} \right] \frac{6}{n}$
2.	$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left[ \left( -2 + \frac{5i}{n} \right)^2 - 3 \right] \frac{5}{n}$
3.	$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left[ 3\left(1 + \frac{5i}{n}\right) - 4 \right] \frac{5}{n}$
4.	$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left[ \left( -2 + \frac{6i}{n} \right)^3 \right] \frac{6}{n}$
5.	$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left[ \sqrt{\left( -2 + \frac{2i}{n} \right)^2 + 1} \right] \frac{2}{n}$
6.	$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left[ 5\left(2 + \frac{4i}{n}\right) + 7 \right] \frac{4}{n}$
7.	$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left[ 6\left(\frac{4i}{n}\right)^2 - 2 \right] \frac{4}{n}$
8.	$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left[ \left(1 + \frac{2i}{n}\right)^3 - 1 \right] \frac{2}{n}$

**Apply Your Understanding of Definite Integrals**

In the chart below, a definite integral has been provided for you. If a definite integral has been provided, write the corresponding limit of a Riemann sum.

	Definite Integral	Limit of Riemann Sum
1.	$\int_0^6 \sqrt{2x+1} dx$	
2.	$\int_{-2}^3 x^2 - 3 dx$	
3.	$\int_1^6 3x - 4 dx$	
4.	$\int_{-2}^4 x^3 dx$	
5.	$\int_{-2}^0 \sqrt{x^2+1} dx$	
6.	$\int_2^6 5x + 7 dx$	
7.	$\int_0^4 6x^2 - 2 dx$	
8.	$\int_1^3 4x^3 - 1 dx$	

Match the following Definite Integrals with summation notation Riemann Sums

AP CALCULUS TEACHING STRATEGIES

Limit of Riemann Sum Cards (print one set per student pair)	
A.	$\lim_{n \rightarrow \infty} \sum_{j=1}^n \left[ 2 \left( 1 + \frac{4j}{n} \right) + 9 \right] \frac{4}{n}$
B.	$\lim_{n \rightarrow \infty} \sum_{j=1}^n \left[ 2 \left( -1 + \frac{4j}{n} \right) + 9 \right] \frac{4}{n}$
C.	$\lim_{n \rightarrow \infty} \sum_{j=1}^n \sqrt{\left( 1 + \frac{4j}{n} \right)^2 + 9} \frac{4}{n}$
D.	$\lim_{n \rightarrow \infty} \sum_{j=1}^n \left( 1 + \frac{2j}{n} \right)^2 + 9 \frac{2}{n}$
E.	$\lim_{n \rightarrow \infty} \sum_{j=1}^n \left[ 2 \left( \frac{2j}{n} \right) + 9 \right] \frac{2}{n}$
F.	$\lim_{n \rightarrow \infty} \sum_{j=1}^n \left[ 2 \left( \frac{4j}{n} \right) + 9 \right] \frac{4}{n}$
G.	$\lim_{n \rightarrow \infty} \sum_{j=1}^n \sqrt{\left( \frac{4j}{n} \right)^2 + 9} \frac{4}{n}$
H.	$\lim_{n \rightarrow \infty} \sum_{j=1}^n \left[ \left( \frac{4j}{n} \right)^2 + 9 \right] \frac{4}{n}$
I.	$\lim_{n \rightarrow \infty} \sum_{j=1}^n \left[ \left( 2 + \frac{2j}{n} \right)^2 + 9 \right] \frac{2}{n}$
J.	$\lim_{n \rightarrow \infty} \sum_{j=1}^n \left[ \left( 1 + \frac{4j}{n} \right)^2 + 9 \right] \frac{4}{n}$

AP CALCULUS TEACHING STRATEGIES

Definite Integral Cards (print one set per student pair)	
Definite Integral #1	Definite Integral #6
$\int_0^4 \sqrt{x^2 + 9} dx$	$\int_0^4 x^2 + 9 dx$
Definite Integral #2	Definite Integral #7
$\int_1^5 \sqrt{x^2 + 9} dx$	$\int_1^5 2x + 9 dx$
Definite Integral #3	Definite Integral #8
$\int_1^5 x^2 + 9 dx$	$\int_{-1}^3 2x + 9 dx$
Definite Integral #4	Definite Integral #9
$\int_0^4 2x + 9 dx$	$\int_1^3 x^2 + 9 dx$
Definite Integral #5	Definite Integral #10
$\int_2^4 x^2 + 9 dx$	$\int_0^2 2x + 9 dx$

## AP CALCULUS

## STUDENT HANDOUT

**Check your understanding**

1. The question below is followed by four student responses – three are INCORRECT and only one is CORRECT. Select the correct answer choice, and identify the error in each of the incorrect responses

Question: Which of the following limits is equal to  $\int_3^5 x^4 dx$  ?

Student Response A:  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(3 + \frac{k}{n}\right)^4 \frac{1}{n}$

Student Response B:  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(3 + \frac{k}{n}\right)^4 \frac{2}{n}$

Student Response C:  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(3 + \frac{2k}{n}\right)^4 \frac{1}{n}$

Student Response D:  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(3 + \frac{2k}{n}\right)^4 \frac{2}{n}$

2. Look back at choice A. What is the definite integral associated with that limit expression?