Calculus 30L - Integration by Trigonometric Substitution

Consider:

$$\int \sqrt{a^2 - x^2}\,dx, \quad \int \sqrt{x^2 - a^2}\,dx, \quad \int \sqrt{x^2 + a^2}\,dx$$

(where a is a constant.)

*For Integrals of these forms, substituting Trigonometric functions allows us to eliminate radicals, making integration easier

Verification: $\sqrt{1 - x^2}$

Note:

- Format is in terms of theta, not x anymore (later we will turn this back to x)
- This works for values other than "1" as well

$$\int \sqrt{a^2 - x^2}\,dx \qquad x = a\sin\theta$$

$dx =$

For $\sqrt{a^2 - x^2}$, use $x = a\sin\theta$

For $\sqrt{a^2 + x^2}$, use $x = a\tan\theta$

For $\sqrt{x^2 - a^2}$, use $x = a\sec\theta$

$$\int \sqrt{x^2 + a^2}\,dx \qquad x = a\tan\theta$$

$dx =$

$$\int \sqrt{x^2 - a^2}\,dx, \qquad x = a\sec\theta$$

$dx =$

Eg! $\int (4 - x^2)^{\frac{1}{2}}\,dx$

Eg2

$$\int \sqrt{3-x^2}\,dx$$

Try it on your own now!

Exercises

Integrate each of the given functions:

1. $\int \sqrt{16-x^2}\,dx$

2. $\int \frac{3\,dx}{x\sqrt{4-x^2}}$

<u>Double substitution</u>

In these examples it may be necessary to convert from x to theta, then to u, then back to theta, and then back to x

Eg.3 $$\int \frac{1}{x^2\sqrt{1+x^2}}\,dx$$

Another Tricky one

3. $\int \frac{dx}{\sqrt{x^2+2x}}$

Trigonometric Substitution Assignment

Name: ____________________

Due: ___________

$$\int \sqrt{4-x^2}\,dx$$

$$\int \frac{1}{x\sqrt{x^2-1}}\,dx$$

$$\int \frac{1}{x^2\sqrt{3x^2-1}}\,dx$$

$$\int \frac{1}{x}\sqrt{x^2-4}\,dx$$

$$\int \frac{1}{x^2\sqrt{x^2+4}}\,dx$$

$$\int (9-4x^2)^{\frac{1}{2}}\,dx$$