Tips For Integrating…

1. **Direct integration**. If it's something like a *polynomial function* that can be integrated with power rule, you'll be able to integrate directly.
2. **U-substitution**. Look to see if the integrand contains both a function and its derivative (including variable part).
3. **Integration by parts.** (DI Method)- The integrand will usually be a product of one of these forms:
   a. The product of an exponential function (e^x) and a trig function ($\sin x$)
   b. The product of a power function (x^2) and a trig function ($\sin x$)
   c. The product of an exponential function (e^x) and a power function (x^2)
4. **Partial fractions.** The integrand will be a rational function (a fraction in which the numerator and denominator are both polynomial functions). You'll need to factor the denominator.

Part 1 - Decomposing Rational Functions

A rational function integrand that cannot be easily integrated by U-Substitution or by Parts (DI method), can sometimes be integrated easily by decomposing the rational into fractional parts with constant number numerators. The predictable process of decomposition is the basis of this lesson.

$$\int \left[\frac{\text{rational}}{(\text{not usub})}\right] dx$$

Consider this example to demonstrate the notion of partial fractions

Eg Simplify $\frac{1}{(x+2)} + \frac{5}{(x+3)}$

* with no x in numerator, partial fractions are much easier to integrate.

Decomposing a rational fraction is the reverse process!
There exists A and B such that:

**Where the degree of the numerator is less than the degree of the denominator*

$$\frac{f(x)}{(x+a)(x+b)} = \frac{A}{(x+a)} + \frac{B}{(x+b)}$$

A rational function can be decomposed into simpler sums of rationals that have constant numerators.

$$\frac{6x+13}{x^2+5x+6} = \frac{A}{(x+2)} + \frac{B}{(x+3)}$$

Method 1. Multiply to eliminate denominators.

Substitute x = ____ in order to eliminate one variable term, and solve for the other. Use restricted values but in reality we are using a value that is extremely close to , but not exactly the restricted value!

$$6(\quad)+13 = A(\quad+3) + B(\quad+2)$$

3 Substitute $B=\boxed{}$ into augmented equation to solve for A.

$$6x+13 = A(x+3) + 5(x+2)$$

OR

3a repeat step 2 to solve for A.

So

$$\frac{6x+13}{(x+2)(x+3)} = \frac{\boxed{}}{(x+2)} + \frac{\boxed{}}{(x+3)}$$

Method 2.

A + B are constants.

$\boxed{\text{Equating coeficients.}}$

① eliminate denom's.

② expand + gather like terms.

③ organize rt. side into 2 terms...

④ Equate coefficients to get a system of equations.

$$\frac{6x+13}{(x+2)(x+3)} = \frac{A}{(x+2)} + \frac{B}{x+3}.$$

$$6x+13 = A(x+3) + B(x+2)$$

subst.

Eg2.

$$\frac{x}{(2-x)(x+3)} = \frac{A}{(2-x)} + \frac{B}{(x+3)}$$

$$\frac{3}{x^2+x}$$

Partial Fractions

Type 2 Repeated Linear Factors.

Eg 1 $\frac{x+5}{(x+3)^2} \Rightarrow \frac{x+5}{(x+3)(x+3)} = \frac{A}{(x+3)} + \frac{B}{(x+3)^2}$

Eg 2 $\frac{x+5}{(x+3)^3} \Rightarrow \frac{x+5}{(x+3)(x+3)(x+3)} = \frac{A}{(x+3)^1} + \frac{B}{(x+3)^2} + \frac{C}{(x+3)^3}$

Eg2

$$\frac{3x-1}{(x+2)^2} = \frac{A}{x+2} + \frac{B}{(x+2)^2}$$

Eg3

$$\frac{1}{(x-1)^2(x+1)}$$

Eg 4

$$\frac{2x^2-3}{(x-1)^3(x+1)}$$

Recall Integrating 1/x… it is ______________

We use this concept in approaching integrals that are not suitable to be solved for U-Sub because without a variable in the numerator, integration is much simpler.

*Split the integrand into partial fractions then integrate…

Part 2 - Integrating Rationals using Partial Fractions

Integrate using Partial Fractions

$$\int \frac{x+5}{(x+3)^2}\,dx$$

Integrate Using Partial Fractions Method

$$\int \frac{2x^2-3}{(x-1)^3(x+1)}\,dx$$

Integrate using Partial Fractions

1.) $\int \frac{3x}{(2x+1)(x+4)}\,dx$ 2.) $\int \frac{5x^3+7x-9}{(x+1)^4}\,dx$ 3.) $\int \frac{1}{x^2(x-2)}\,dx$

Solutions to previous questions #1-3

1.

$$\frac{3x}{(2x+1)(x+4)} = \frac{-3}{7(2x+1)} + \frac{12}{7(x+4)}$$

2.

$$\boxed{\frac{5}{x+1} - \frac{15}{(x+1)^2} + \frac{22}{(x+1)^3} - \frac{21}{(x+1)^4}}$$

3.

$$= -\frac{1}{4}\ln|x| + \frac{1}{2x} + \frac{1}{4}\ln|x-2| + K$$