

AP CALCULUS DIFFERENTIAL EQUATIONS

1. Oil is leaking from a tanker at a rate of  $R(t) = 500e^{-0.2t}$  gallons per hour, where  $t$  is measured in hours. The amount of oil that has leaked out (in gallons) after 10 hours is:
- a) 2140   b) 2150   c) 2160   d) 2170   e) 2180
2. If the graph  $y = f(x)$  contains the point  $(0, 1)$  and if  $\frac{dy}{dx} = \frac{x \sin(x^2)}{y}$  then  $f(x)$  is
- a)  $\sqrt{2 - \cos(x^2)}$    b)  $\sqrt{2} - \cos x^2$    c)  $2 - \cos x^2$    d)  $\cos x^2$   
 e)  $\sqrt{2 - \cos x}$
3. Solve the following  $\frac{dy}{dx} = \frac{-x}{y}$  given the initial condition  $y(0) = 2$ .
4. The point  $(1, 9)$  lies on the graph of an equation  $y = f(x)$  for which  $\frac{dy}{dx} = 4x\sqrt{y}$  where  $y \geq 0$ . When  $x = 0$ ,  $y = ?$
- a) 0   b) 1   c) 2   d) 3   e) 4
5. If the graph of  $y = f(x)$  defined for all  $x \geq 0$  contains the point  $(0, 1)$  has  $\frac{dy}{dx} = 3\sqrt{xy}$  and  $f(x) > 0$ , for all  $x$ , then  $f(x) =$
- a)  $(x^3 + 1)^2$    b)  $(x^{\frac{3}{2}} + 1)^2$    c)  $(x^{\frac{1}{2}} + 1)^2$    d)  $(x^3 - 1)^2$   
 e)  $(x^{\frac{3}{2}} - 1)^2$
6. Let  $f$  be a function with  $f(1) = 4$  such that the graph of  $f$  has a slope given by  $\frac{3x^2 + 1}{2y}$ .
- a) Find the slope at  $x = 1$
- b) Write the equation of the tangent line at  $x = 1$  and use it to approximate  $f(1.2)$
- c) Find  $f(x)$  by solving the equation  $\frac{dy}{dx} = \frac{3x^2 + 1}{2y}$  with the initial condition  $f(1) = 4$
- d) Use your solution from part (c) to find  $f(1.2)$

7.  $V(t)$  is the velocity of a skydiver at time  $t$  seconds  $t \geq 0$ . After the parachute opens the velocity satisfies the differential equation  $\frac{dv}{dt} = -2v - 32$  with the initial condition  $v(0) = -50$ .
- Use separation of variables to find an expression for  $v$  in terms of  $t$ .
  - Terminal velocity is defined as  $\lim_{t \rightarrow \infty} v(t)$ . Find the terminal velocity to the nearest foot.
  - It is safe to land when her speed is 20 feet per second. At what time does she reach this speed?
8. Consider the differential equation  $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$ . Find a solution for  $y = f(x)$  to the differential equation satisfying  $f(0) = 1$ .
9. A solution of the equation  $\frac{dy}{dx} + 2xy = 0$  that contains the point  $(0, e)$  is:
- $y = e^{1-x^2}$
  - $y = e^{1+x^2}$
  - $y = e^{1-x}$
  - $y = e^{1+x}$
  - $y = e^{x^2}$

**Answers**

- C
- A
- $y = \sqrt{-x^2 + 4}$
- E
- B
- a)  $\frac{1}{2}$     b)  $y = 4.1$     c)  $f(x) = \sqrt{x^3 + x + 14}$     d)  $f(1.2) = 4.114$
- a)  $V = -34e^{-2t} - 16$     b)  $-16$     c)  $t = 1.070s$
- $y = \frac{1}{2} \ln(2x^3 + e)$
- A