Slope Fields

Graphical Representations of Solutions to Differential Equations

A **slope field** is a pictorial representation of all of the possible solutions to a given differential equation.

Remember that a differential equation is the first derivative of a function, $f'(x)$ or $\frac{dy}{dx}$. Thus, the solution to a differential equation is the function, *f*(*x*) or *y*.

There is an infinite number of solutions to the differential equation $\frac{dy}{dx} = x + 3$. Show your work and explain why.

For the AP Exam, you are expected to be able to do the following four things with slope fields:

1. ______________________________

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2. ______________________________

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3. ______________________________

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4. ______________________________

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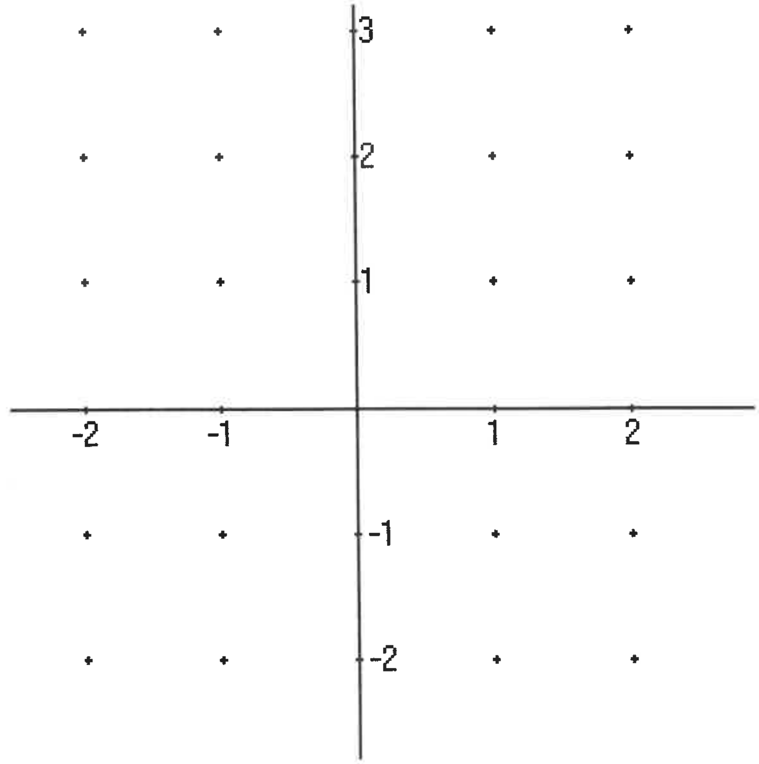
#1 Sketch a slope field for a given differential equation.

Given the differential equation below, compute the slope for each point Indicated on the grid to the right.

Then, make a small mark that approximates the slope through the point.

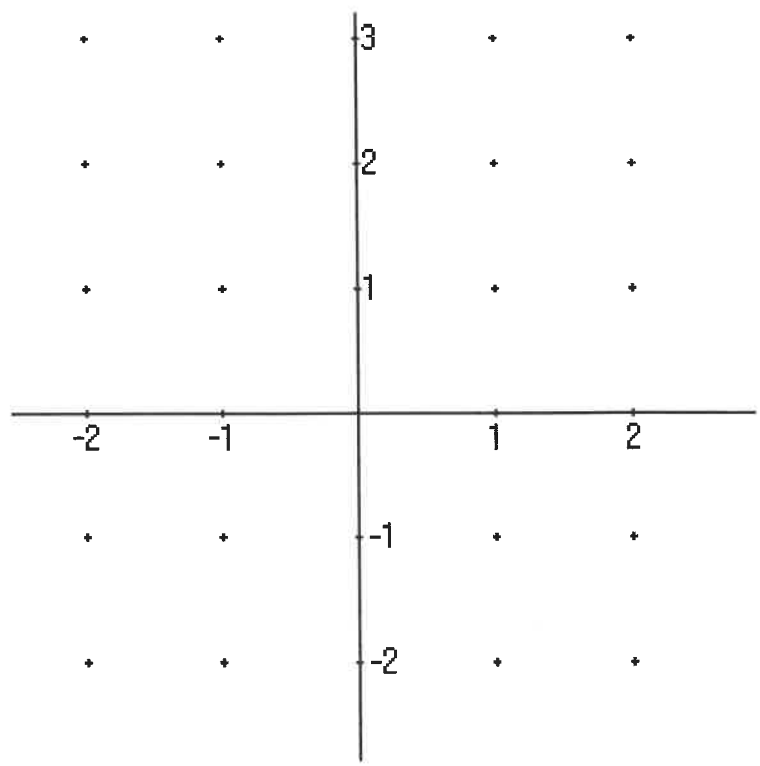
$$\frac{dy}{dx} = x - 1$$



Given the differential equation below, compute the slope for each point Indicated on the grid to the right.

Then, make a small mark that approximates the slope through the point.

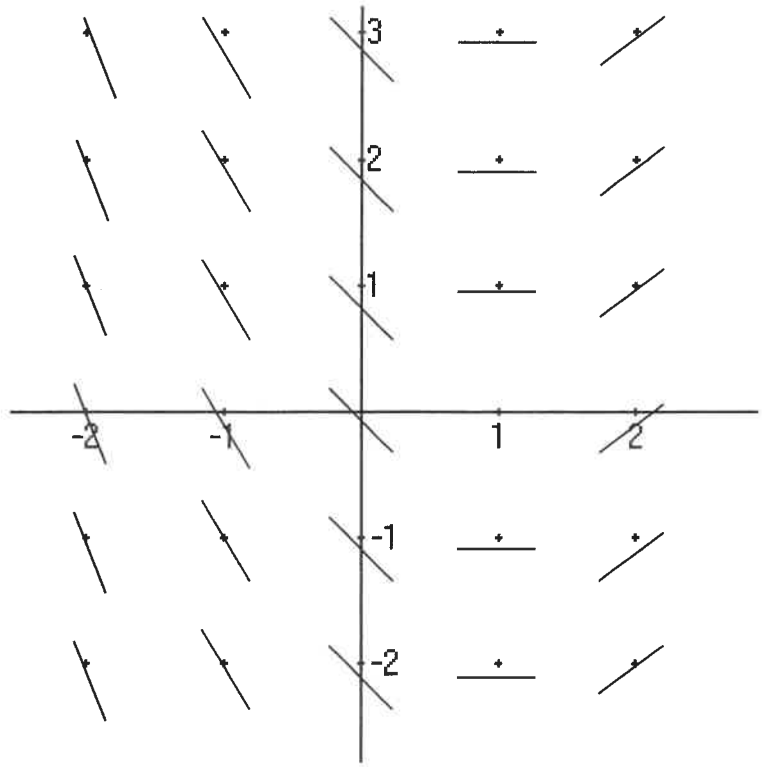
$$\frac{dy}{dx} = x - y$$



#2 Given a slope field, sketch a solution curve through a given point.

To the right is pictured the slope field that you developed for the differential equation on the previous page.

$$\frac{dy}{dx} = x - 1$$



Sketch the solution curve through the point (1, -1).

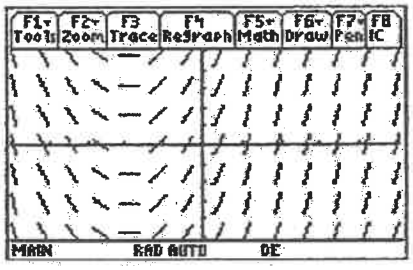
To do this, you find the point and then follow the slopes as you connect the lines.

#3 Match a slope field to a differential equation.

Since the slope field represents all of the particular solutions to a differential equation, and the solution represents the ANTIDERIVATIVE of a differential equation, then the slope field should take the shape of the antiderivative of *dy*/*dx*.

Match the slope fields to the differential equations on the next page.

A.



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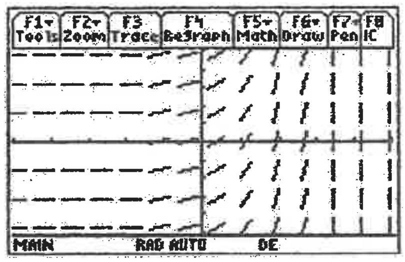

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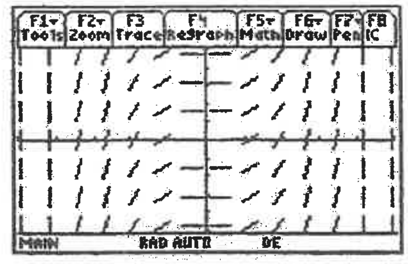
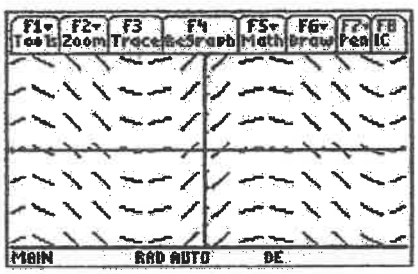


D.



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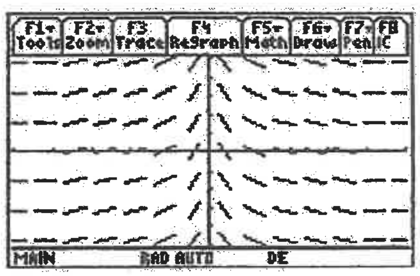

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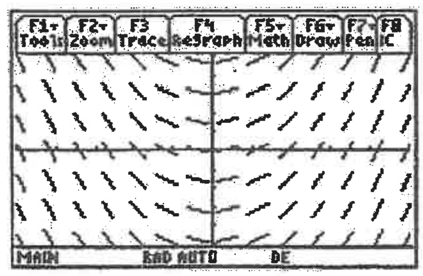

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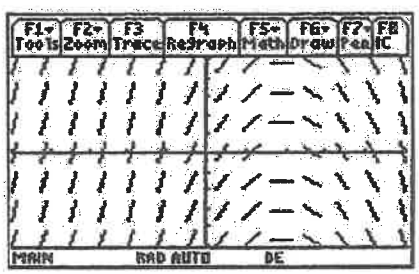

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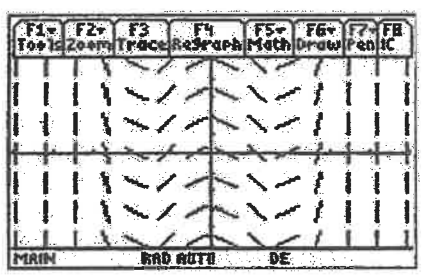
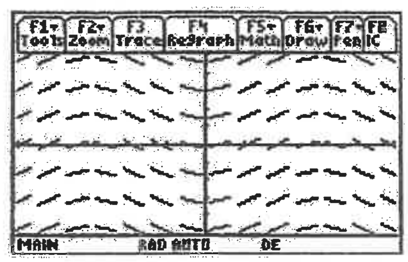
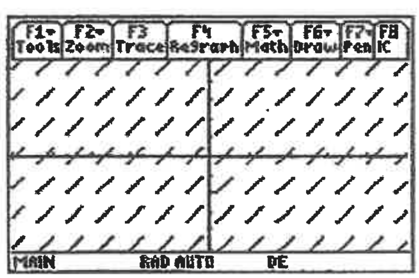


I.



J.



Separate the variables and find the general solution to each differential equation below to determine what the slope field should look like for each. Then, match to the graphs of slope fields on the previous page.

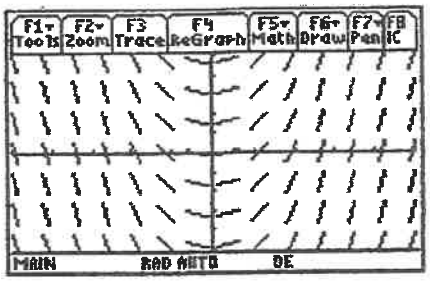
	1. $\frac{dy}{dx}=\sin x$		2. $\frac{dy}{dx}=2x+4$
	3. $\frac{dy}{dx}=e^x$		4. $\frac{dy}{dx}=2$
	5. $\frac{dy}{dx}=x^3-3x$		6. $\frac{dy}{dx}=2\cos x$
	7. $\frac{dy}{dx}=4-2x$		8. $\frac{dy}{dx}=x$
	9. $\frac{dy}{dx}=x^2$		10. $\frac{dy}{dx}=-\frac{1}{x}$

#4 Match a slope field to a solution to a differential equation.

When given a slope field and a solution to a differential equation, then the slope field should look like the solution, or *y*.

Match the slope fields below to the solutions on the next page.

A.



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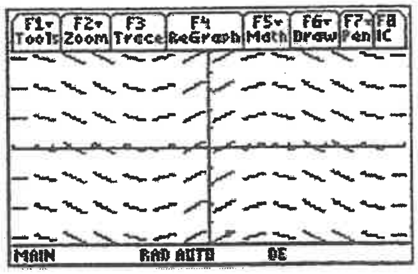

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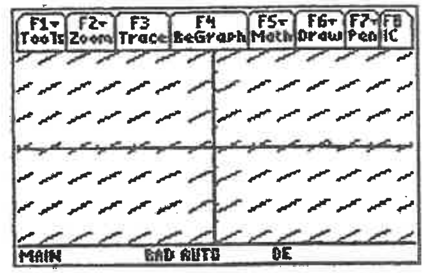

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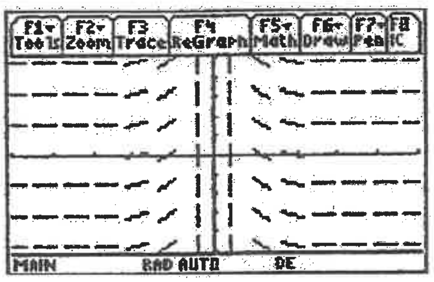

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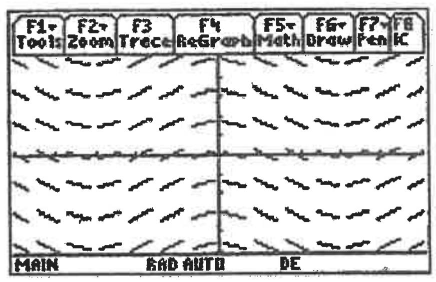

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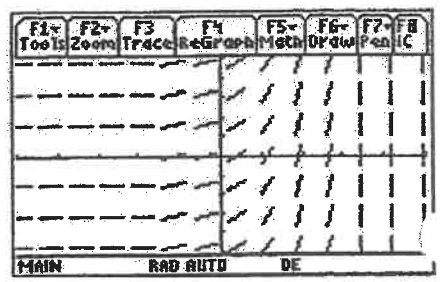

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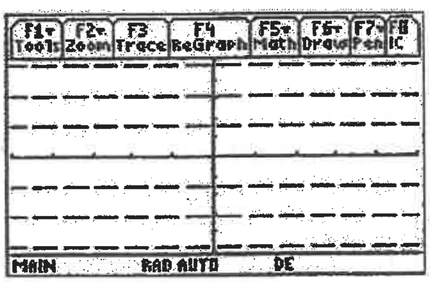

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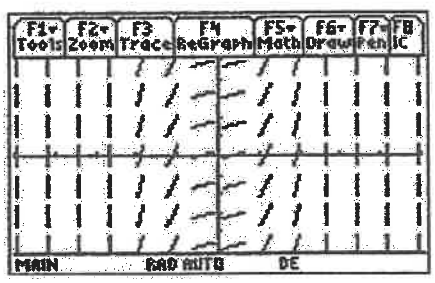
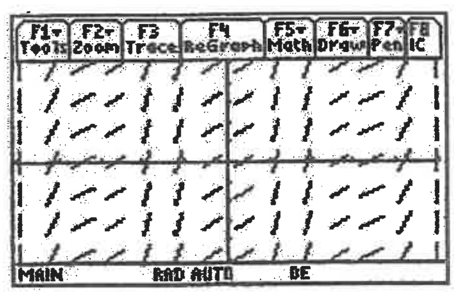


I.



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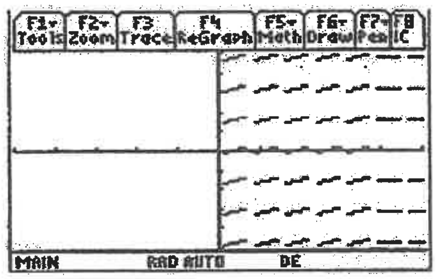
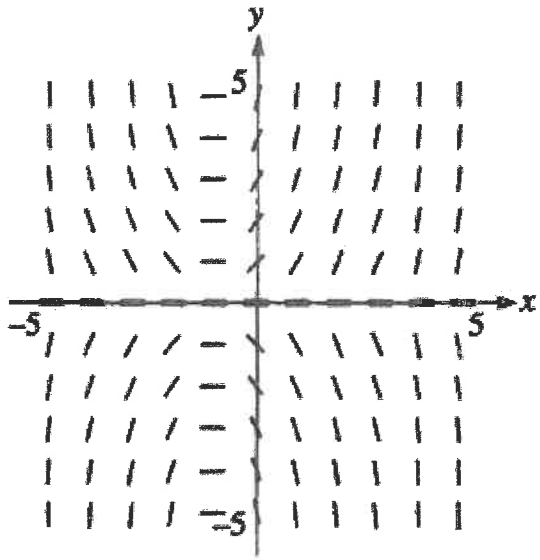


	1. $y = x$		2. $y = x^2$
	3. $y = e^x$		4. $y = \frac{1}{x^2}$
	5. $y = x^3$		6. $y = \sin x$
	7. $y = \cos x$		8. $y = \sqrt{x}$
	9. $y = 1$		10. $y = \tan x$

Shown below is a slope field for which of the following differential equations? Explain your reasoning for each of the choices below.



(A) $\frac{dy}{dx} = xy$

(B) $\frac{dy}{dx} = xy - y$

(C) $\frac{dy}{dx} = xy + y$

(D) $\frac{dy}{dx} = xy + x$

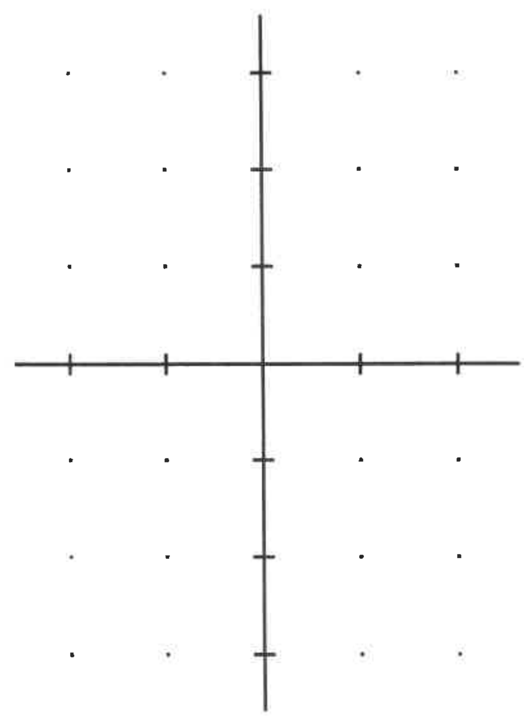
(E) $\frac{dy}{dx} = (x+1)^3$

Consider the differential equation $\frac{dy}{dx} = \frac{x}{y}$ to answer the following questions.

a. On the axes below, sketch a slope field for the equation.

b. Sketch a solution curve that passes through the point $(0, -1)$ on your slope field.

c. Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(0) = -1$.

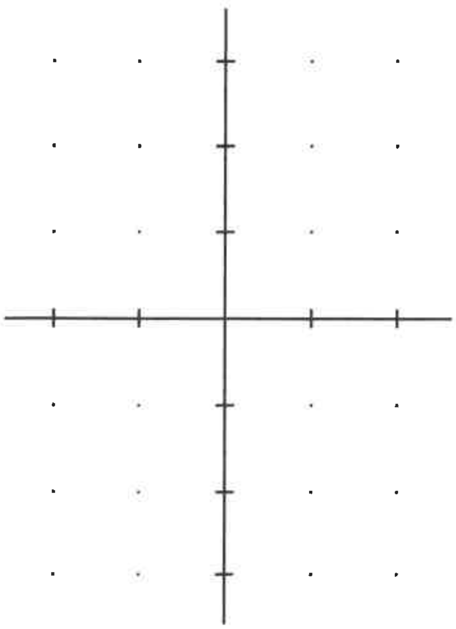


Name________________________________Date__________________Class__________

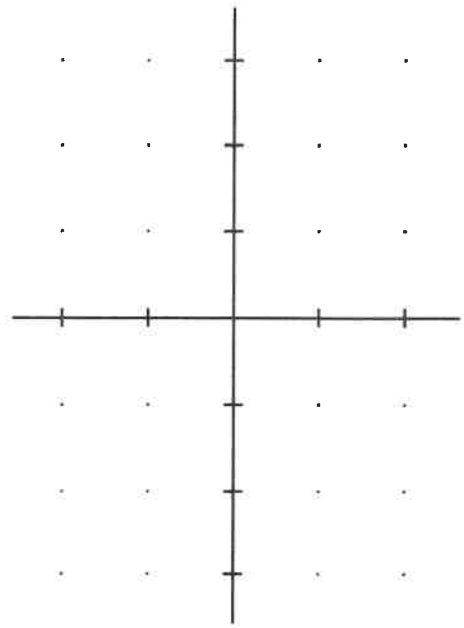
Day #60 Homework

For the indicated points on each grid, draw the slope field for the given differential equation.

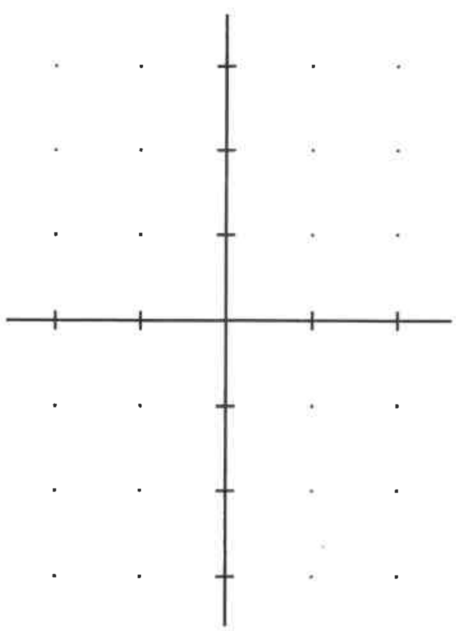
1. $\dfrac{dy}{dx} = x + y$



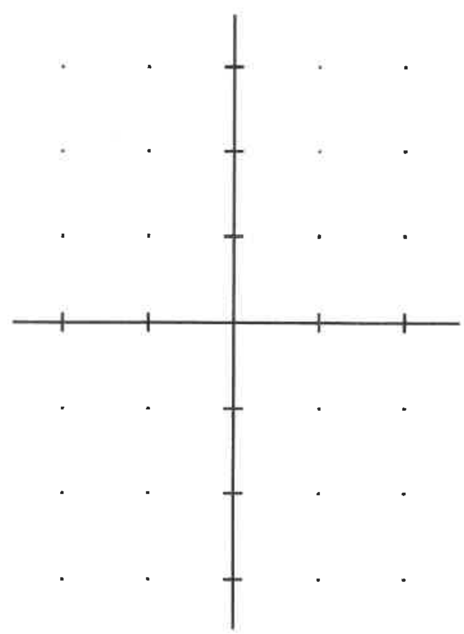
2. $\dfrac{dy}{dx} = -\dfrac{y}{x}$



3. $\dfrac{dy}{dx} = x + 1$

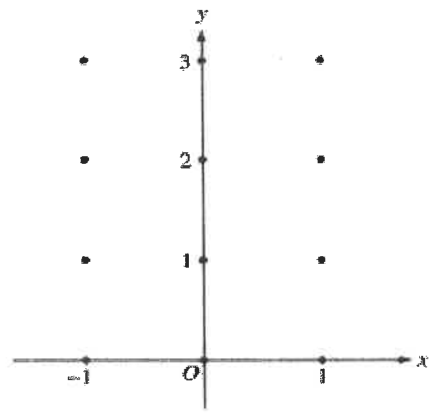


4. $\dfrac{dy}{dx} = \dfrac{1}{x+1}$



2004 AP® CALCULUS AB

Question 6



Consider the differential equation $\frac{dy}{dx} = x^2(y-1)$.

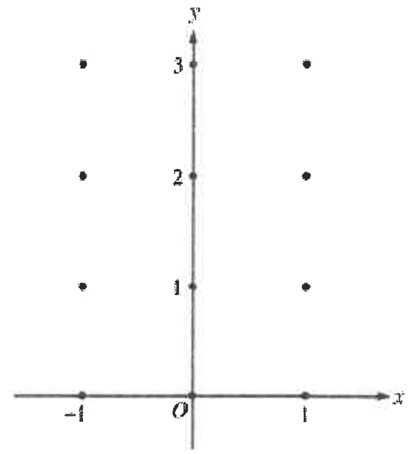
(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the pink test booklet.)

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are positive.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.

2004 AP® CALCULUS AB
Question 5 (Form B)

Consider the differential equation $\frac{dy}{dx} = x^4(y-2)$.



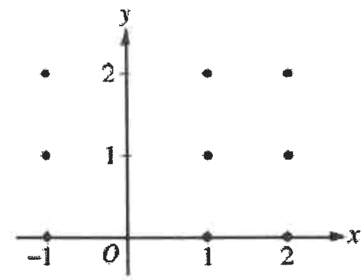
(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the test booklet.)

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are negative.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 0$.

Consider the differential equation $\frac{dy}{dx} = \frac{y-1}{x^2}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
(Note: Use the axes provided in the exam booklet.)

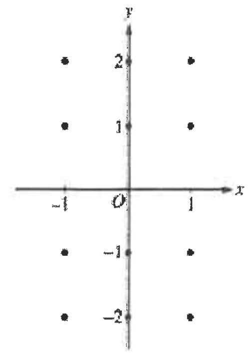


(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$.

(c) For the particular solution $y = f(x)$ described in part (b), find $\lim_{x \to \infty} f(x)$.

2010 AP® CALCULUS AB (Form B)
Question 5

Consider the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$.



(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and for $-1 < x < 1$, sketch the solution curve that passes through the point $(0, -1)$.

(Note: Use the axes provided in the exam booklet.)

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane for which $y \neq 0$. Describe all points in the xy-plane, $y \neq 0$, for which $\frac{dy}{dx} = -1$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -2$.

Solving Differential Equations
Examples of Variable Separable Differential Equations

Given below are differential equations with given initial condition values. Find the particular solution for each differential equation.

1. $\frac{dy}{dx} = 6x^2 + 6x + 2$ and $f(-1) = 2$	2. $\frac{dy}{dx} = \frac{1 + 12x^{3/2}}{2\sqrt{x}}$ and $f(0) = 2$
3. $\frac{dy}{dx} = \frac{x^2 + 2x}{2y}$ and $f(0) = 2$	4. $\frac{dy}{dx} = \frac{x + 2}{y}$ and $f(1) = -3$

5. $\frac{dy}{dx} = x^4(y-2)$ and $f(0) = 0$	6. $\frac{dy}{dx} = \frac{y-1}{x^2}$ and $f(2) = 0$

2000 AP Calculus AB
Question 6

Consider the differential equation $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$.

(a) Find a solution $y = f(x)$ to the differential equation satisfying $f(0) = \frac{1}{2}$.

(b) Find the domain and range of the function f found in part (a).

The acceleration of a particle moving along the x – axis at time t is given by $a(t) = 6t - 2$. If the velocity is 25 when $t = 3$ and the position is 10 when $t = 1$, then the position $x(t) =$

A. $9t^2 + 1$

B. $3t^2 - 2t + 4$

C. $t^3 - t^2 + 4t + 6$

D. $t^3 - t^2 + 9t - 20$

E. $36t^3 - 4t^2 - 77t + 55$

A particle moves along the x-axis so that, at any time $t \geq 0$, its acceleration is given by $a(t) = 6t + 6$. At time $t = 0$, the velocity of the particle is –9 and its position is –27.

a. Find $v(t)$, the velocity of the particle at any time t.

b. Find the net distance traveled by the particle over the interval [0, 2].

c. Find the total distance traveled by the particle over the interval [0, 2].

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Day #59 Homework

AP® CALCULUS AB 2001 Question 6

The function f is differentiable for all real numbers. The point $\left(3, \frac{1}{4}\right)$ is on the graph of $y = f(x)$, and the slope at each point (x, y) on the graph is given by $\frac{dy}{dx} = y^2(6 - 2x)$.

(a) Find $\frac{d^2y}{dx^2}$ and evaluate it at the point $\left(3, \frac{1}{4}\right)$.

(b) Find $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = y^2(6 - 2x)$ with the initial condition $f(3) = \frac{1}{4}$.

Consider the differential equation $\frac{dy}{dx} = \frac{3-x}{y}$.

(a) Let $y = f(x)$ be the particular solution to the given differential equation for $1 < x < 5$ such that the line $y = -2$ is tangent to the graph of f. Find the x-coordinate of the point of tangency, and determine whether f has a local maximum, local minimum, or neither at this point. Justify your answer.

(b) Let $y = g(x)$ be the particular solution to the given differential equation for $-2 < x < 8$, with the initial condition $g(6) = -4$. Find $y = g(x)$.