The Squeeze Theorem

If $f(x) \le g(x) \le h(x)$ for x near a, except possibly at a, and

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L\,,$$

then

$$\lim_{x \to a} g(x) = L.$$

Graph:

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then $\lim_{x \to a} g(x) = L$.

Squeeze Theorem

If $\lim_{x \to a} f(x) \leq \lim_{x \to a} g(x) \leq \lim_{x \to a} h(x)$

and if $\lim_{x \to a} f(x) = L = \lim_{x \to a} h(x)$,

then $\lim_{x \to a} g(x) = L$.

A theorem by any other name will apply as well?

Sandwich Theorem

Pinching Theorem

Squeeze Play Theorem

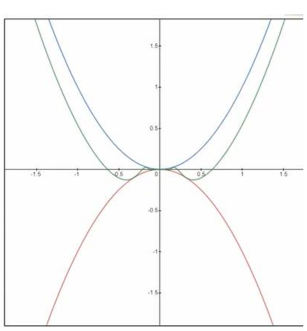
Flyswatter Theorem

And others…

Evaluate the limit using squeeze theorem.

$$\lim_{x \to 0} \left(x^2 \cos\left(\frac{1}{x}\right) \right)$$

Methods? Why not properties?



Takeaways…

Mathematicians are doing us a favor when they don't name theorems after people.

What does the squeeze theorem do? It squeezes a function between two better known or easier to work with functions.

Known minimums and maximums can come in really handy.

The function $f(x) = \cos\left(\frac{1}{x}\right)\sin\left(\frac{x^3-1}{x^4}\right)$ is bounded from below by $g(x) = -\sqrt{1.11 - x^2}$ and from above by $h(x) = \sqrt{1.11 - x^2}$ on the interval [-1, 1]. Is there enough information to determine $\lim_{x\to 0}\left(\cos\left(\frac{1}{x}\right)\sin\left(\frac{x^3-1}{x^4}\right)\right)$?

What does the squeeze theorem do? It squeezes a function between two better known or easier to work with functions <u>that have the same limit at the value in question</u>.

Known bounding functions are handy but not definitive.

The traditional proof uses (some really neat) areas and trigonometry in the unit circle to show that $\sin x \le x \le \tan x$ and that $\cos x \le \frac{\sin x}{x}$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$. Use this information to find: $\lim_{x\to 0}\frac{\sin x}{x}$.