MVT AP Exam Question Practice Asn't

x	0	4	8	12	16
$f(x)$	8	0	2	10	1

12. The table above gives selected values for the differentiable function f. In which of the following intervals must there be a number c such that $f'(c) = 2$?

(A) $(0, 4)$ (B) $(4, 8)$ (C) $(8, 12)$ (D) $(12, 16)$

20. Let g be a twice-differentiable, increasing function of t. If $g(0) = 20$ and $g(10) = 220$, which of the following must be true on the interval $0 < t < 10$?

(A) $g'(t) = 0$ for some t in the interval.

(B) $g'(t) = 20$ for some t in the interval.

(C) $g''(t) = 0$ for some t in the interval.

(D) $g''(t) > 0$ for all t in the interval.

26. Let f be the function with $f(0) = \frac{1}{\pi^2}$, $f(2) = \frac{1}{\pi^2}$, and derivative given by $f'(x) = (x + 1)\cos(\pi x)$. How many values of x in the open interval $(0, 2)$ satisfy the conclusion of the Mean Value Theorem for the function f on the closed interval $[0, 2]$?

(A) None

(B) One

(C) Two

(D) More than two

x	$f(x)$
–1	–30
0	–2
3	10
5	18

21. The table above gives selected values for a twice-differentiable function f. Which of the following must be true?

(A) f has no critical points in the interval $-1 < x < 5$.

(B) $f'(x) = 8$ for some value of x in the interval $-1 < x < 5$.

(C) $f'(x) > 0$ for all values of x in the interval $-1 < x < 5$.

(D) $f''(x) < 0$ for all values of x in the interval $-1 < x < 5$.

(E) The graph of f has no points of inflection in the interval $-1 < x < 5$.

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$$\frac{f(12)-f(8)}{12-8}=\frac{10-2}{4}=2$$

So by the MVT, there must be a number c such that $f'(c)=\dfrac{f(b)-f(a)}{b-a}$

where $a<c<b$.

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g' is differentiable $\Rightarrow$ g' is continuous so The Mean Value Theorm guarantees that $g'(t)=20$ for some t in the interval $0<t<10$ because the average rate of change is

$$\frac{g(10)-g(0)}{10}=\frac{220-20}{10}=20$$

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(A) None

(B) One

[(C)] Two

(D) More than two

$$\frac{f(2)-f(0)}{2}=0\Rightarrow f'(c)=0$$
$$f'(c)=(c+1)\cos(\pi c)=0$$
$$c+1=0\Rightarrow c=-1 \text{ not in } (0,2)$$
$$\cos(\pi c)=0\Rightarrow c=\frac{1}{2},\frac{3}{2}$$

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Mean Value Thereom: average rate of change $\dfrac{18-(-30)}{5-(-1)}=\dfrac{48}{6}=8=f'(c)$