IVT Notes

Intermediate Value Theorem

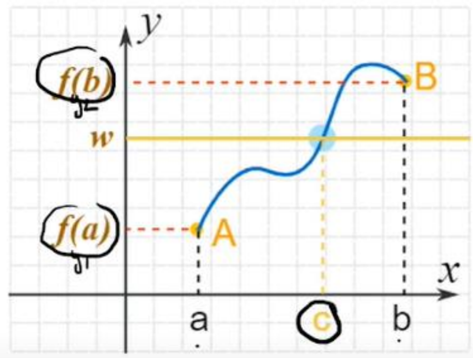
In math, theorems are rules that have hypotheses and conclusions.
If the hypotheses hold true, then the conclusion(s) naturally follow.

Hypothesis 1 - If f(x) is continuous on a closed interval [a,b]
Hypothesis 2 - If W is between f(a) and f(b)
Then;
Conclusion- there will exist x=c between x=a and x=b where f(c) =W

Here is the Intermediate Value Theorem stated more formally:

When:

- The curve is the function $y = f(x)$,
- which is continuous on the interval [a, b],
- and w is a number between f(a) and f(b),



Then ...

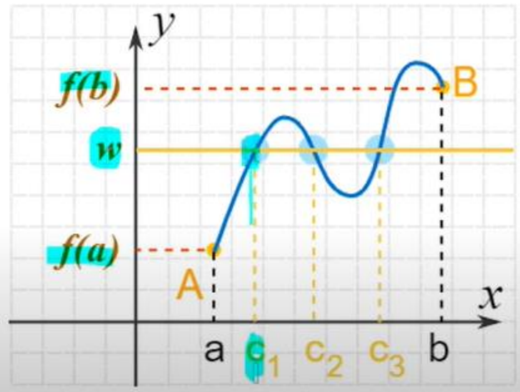
... there must be at least one value **c** within [a, b] such that **f(c) = w**

Summary: IVT simply says that a continuous function takes on ALL values between f(a) and f(b)

At Least One

It also says "at least one value c", which means we **could** have more.

Here, for example, are 3 points where f(x)=w:



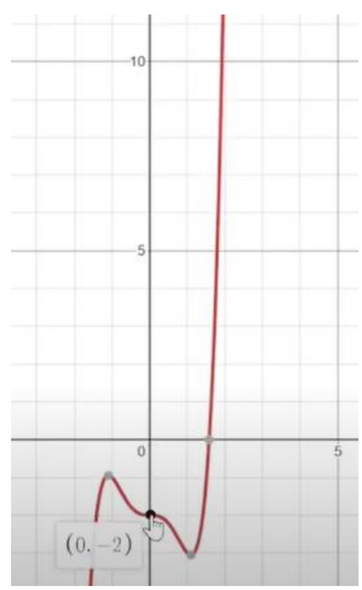
How Is This Useful?

Whenever we can show that:

- there is a point above some line
- and a point below that line, and
- that the curve is continuous,

we can then safely say "yes, there is a value somewhere **in between** that is on the line".

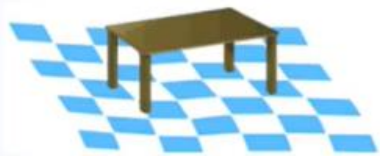
Example: is there a solution to $x^5 - 2x^3 - 2 = 0$ between x=0 and x=2?



AP Calculus AB Mr. Maxwell Name: ___________________

An Interesting Thing!

The Intermediate Value Theorem Can Fix a Wobbly Table



If your table is wobbly because of uneven ground …

… just **rotate the table** to fix it!

The ground must be continuous (no steps such as poorly laid tiles).

Why does this work?

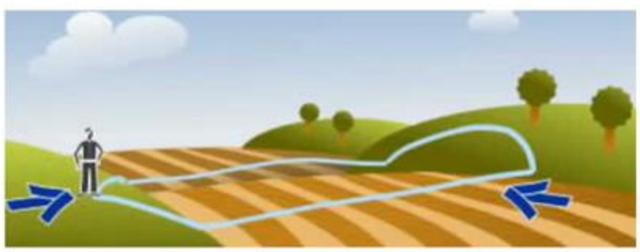
We can always have 3 legs on the ground, it is the 4th leg that is the trouble.

Imagine we are **rotating the table**, and the 4th leg could somehow go into the ground (like sand):

- at some point it will be above the ground
- at another point it will be below the ground

So there must be some point where the 4th leg **perfectly touches the ground** and the table won't

Another One



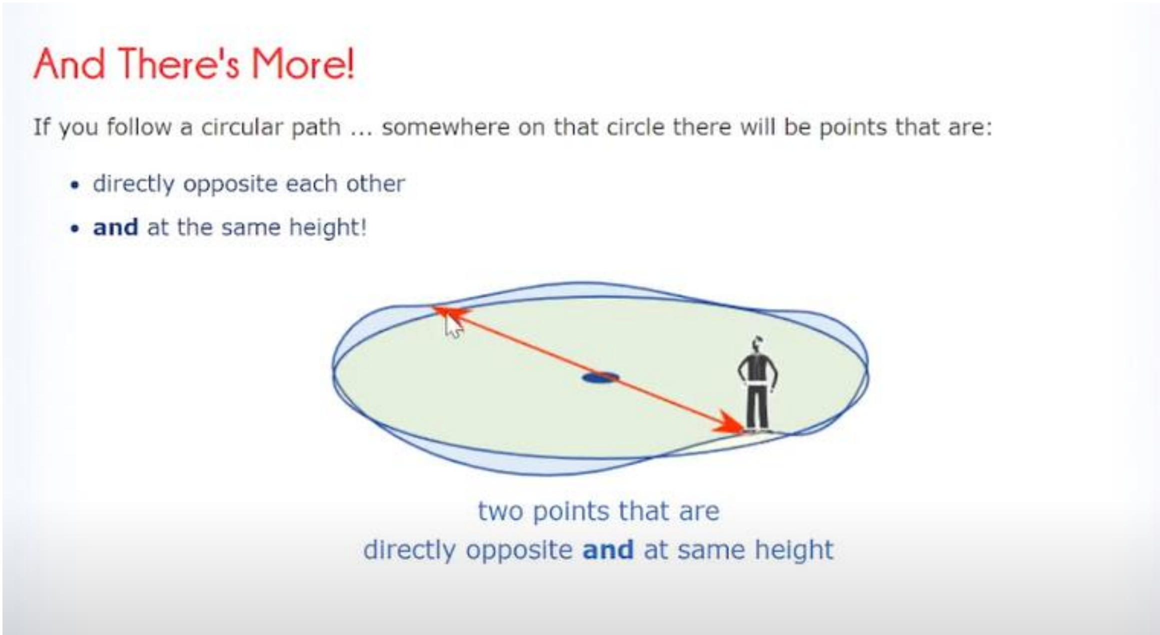
At some point during a round-trip you will be exactly as high as where you started.

(It only works if you don't start at the highest or lowest point.)

The idea is:

- at some point you will be higher than where you started
- at another point you will be lower than where you started

So there must be a point in between where you are **exactly** as high as where you started.



20. Let f be a continuous function such that $f(1) = 7$ and $f(7) = 1$. Let g be the function given by $g(x) = f(x) - x$. Explain why there must be a value c for $1 < c < 7$ such that $g(c) = 0$.

21. The function f is continuous on the closed interval $[1, 3]$ and has values that are given in the table below.

x	1	2	3
$f(x)$	2	k	3

The equation $g(x) = 1$ must have at least two intersections with f in the interval $[1, 3]$ if $k =$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

22. Suppose f is continuous on the closed interval $[0,4]$ and suppose $f(0) = 1, f(1) = 2, f(2) = 0, f(3) = -3, f(4) = 3$. Which of the following statements about the zeros of f on $[0,4]$ is always true?

(A) f has exactly one zero on $[0, 4]$.

(B) f has more than one zero on $[0, 4]$.

(C) f has more than two zeros on $[0, 4]$.

(D) f has exactly two zeros on $[0, 4]$.

(E) None of the statements above is true.

23. Is there a number that is exactly 1 less than its fourth power? Prove, using IVT, that there are 2 solutions.

24. Is any real number exactly 2 more than its cube? Prove or disprove using IVT.