

Practice Problems

Try some of the problems below. If you get stuck, don't worry! There are hints on the next page! But do try without looking at them first, chances are you won't get hints on your exam.

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| 1. $\int_{-1}^1 3x^2\sqrt{x^3+5} dx$ | 12. $\int \frac{x}{(x^2+1)^2} dx$ | 23. $\int_{-3}^0 -\frac{8x}{(2x^2+3)^2} dx$ |
| 2. $\int x^3(2+x^4)^5 dx$ | 13. $\int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} dx$ | 24. $\int e^{\cos(t)} \sin(t) dt$ |
| 3. $\int_0^7 \sqrt{4+3x} dx$ | 14. $\int e^x \sin(e^x) dx$ | 25. $\int_0^1 \frac{16x}{(4x^2+4)^2} dx$ |
| 4. $\int \frac{1}{(1-6t)^4} dt$ | 15. $\int_{-1}^0 \frac{8x}{(4x^2+1)^2} dx$ | 26. $\int \frac{\tan^{-1}(x)}{1+x^2} dx$ |
| 5. $\int_0^{\sqrt{\pi}} x \cos(x^2) dx$ | 16. $\int \frac{x}{x^2+1} dx$ | 27. $\int_{-1}^0 18x^2(3x^3+3)^2 dx$ |
| 6. $\int \frac{\sec(1/x)}{x^2} dx$ | 17. $\int_0^1 -12x^2(4x^3-1)^3 dx$ | 28. $\int \frac{\sin(\ln(x))}{x} dx$ |
| 7. $\int_{1/6}^{1/2} \csc(\pi t) \cot(\pi t) dt$ | 18. $\int \sec(2\theta) \tan(2\theta) d\theta$ | 29. $\int_0^1 -\frac{8x}{(4x^2+2)^2} dx$ |
| 8. $\int x^2(x^3+5)^9 dx$ | 19. $\int_{-1}^2 6x(x^2-1)^2 dx$ | 30. $\int \frac{e^x}{e^x+1} dx$ |
| 9. $\int_0^1 xe^{-x^2} dx$ | 20. $\int \sqrt{x} \sin(1+x^{3/2}) dx$ | 31. $\int \frac{\cos(\pi/x)}{x^2} dx$ |
| 10. $\int (3t+2)^{2.4} dt$ | 21. $\int_0^1 \frac{24x}{(4x^2+4)^2} dx$ | 32. $\int \frac{\sin(x)}{1+\cos^2(x)} dx$ |
| 11. $\int_0^{\pi/2} \cos(x) \sin(\sin(x)) dx$ | 22. $\int (1+\tan(\theta))^5 \sec^2(\theta) d\theta$ | 33. $\int \frac{1}{\cos^2(t)\sqrt{1+\tan(t)}} dt$ |

Challenge Problems

Below are some harder problems that require a little more thinking/algebraic manipulation to make the substitutions work.

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| 1. $\int_0^1 \frac{x}{\sqrt{x+1}} dx$ | 5. $\int \frac{x^2}{\sqrt{1-x}} dx$ | 9. $\int \frac{3x-1}{x^2+10x+28} dx$ |
| 2. $\int \frac{1}{2x^2-12x+26} dx$ | 6. $\int x^3\sqrt{x^2+1} dx$ | 10. $\int_0^4 \frac{x}{\sqrt{1+2x}} dx$ |
| 3. $\int \frac{x}{1+x^4} dx$ | 7. $\int \frac{1}{\sqrt{21-4x-x^2}} dx$ | 11. $\int_{-1}^1 \frac{\sin(x)}{1+x^2} dx$ |
| 4. $\int (x+3)\sqrt{x-1} dx$ | 8. $\int_{-\pi/2}^{\pi/2} \frac{x^2 \sin(x)}{1+x^6} dx$ | 12. $\int \frac{1}{e^x+1} dx$ |

Hints to Practice Problems

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|-------------------|----------------------------|------------------------|
| 1. $u = x^3 + 5$ | 12. $u = x^2 + 1$ | 23. $u = 2x^2 + 3$ |
| 2. $u = 2 + x^4$ | 13. $u = \sin^{-1}(x)$ | 24. $u = \cos(t)$ |
| 3. $u = 4 + 3x$ | 14. $u = e^x$ | 25. $u = 4x^2 + 4$ |
| 4. $u = 1 - 6t$ | 15. $u = 4x^2 + 1$ | 26. $u = \tan^{-1}(x)$ |
| 5. $u = x^2$ | 16. $u = x^2 + 1$ | 27. $u = 3x^3 + 3$ |
| 6. $u = 1/x$ | 17. $u = 4x^3 - 1$ | 28. $u = \ln(x)$ |
| 7. $u = \pi t$ | 18. $u = 2\theta$ | 29. $u = 4x^2 + 2$ |
| 8. $u = x^3 + 5$ | 19. $u = x^2 - 1$ | 30. $u = e^x + 1$ |
| 9. $u = -x^2$ | 20. $u = 1 + x^{3/2}$ | 31. $u = \pi/x$ |
| 10. $u = 3t + 2$ | 21. $u = 4x^2 + 4$ | 32. $u = \cos(x)$ |
| 11. $u = \sin(x)$ | 22. $u = 1 + \tan(\theta)$ | 33. $u = 1 + \tan(t)$ |

Hints to Challenge Problems

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|-------------------------|--|---|
| 1. $x = u - 1$ | 6. $x^2 = u - 1$ | 10. $x = \frac{1}{2}(u - 1)$ |
| 2. Complete the square. | 7. Complete the square. | |
| 3. $u = x^2$ | 8. This is an odd function. | 11. This is an odd function. |
| 4. $u + 4 = x + 3$ | 9. Complete the square,
$x = u - 5$ | 12. $1 = e^x + 1 - e^x$, $u = e^x + 1$ |
| 5. $x = 1 - u$ | | |

Selected Answers.

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|----------------------------------|-------------------------------------|
| 1. $4\sqrt{6} - \frac{16}{3}$ | 8. $\frac{1}{30}(x^3 + 5)^{10} + C$ |
| 2. $\frac{1}{24}(2 + x^4)^6 + C$ | 9. $\frac{e-1}{2e}$ |
| 3. 26 | |

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|---------------------------------------|
| 11. $1 - \cos(1)$ |
| 12. $-\frac{1}{2x^2 + 2} + C$ |
| 13. $\frac{1}{2}(\sin^{-1}(x))^2 + C$ |

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|-------------------------|
| 28. $-\cos(\ln(x)) + C$ |
| 29. $-\frac{1}{3}$ |
| 30. $\ln(e^x + 1) + C$ |

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| 10. $\frac{5}{51}\sqrt[5]{(3t+2)^2}(27t^3 + 54t^2 + 36t + 8) + C$ |
| 11. $1 - \cos(1)$ |

Answers to Challenge Problems

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|---|
| 1. $\frac{4 - 2\sqrt{2}}{3}$ |
| 2. $\frac{1}{4}\tan^{-1}\left(\frac{x-3}{2}\right) + C$ |
| 3. $\frac{1}{2}\tan^{-1}(x^2) + C$ |