

# PreCalculus 3D - Proving Identities Assignment Name: \_\_\_\_\_

Prove that each of the following equations is an identity.

1.  $\sin \theta (\csc \theta - \sin \theta) = \cos^2 \theta$
2.  $\tan \theta (\cot \theta + \tan \theta) = \sec^2 \theta$
3.  $\csc \theta (\csc \theta - \sin \theta) = \cot^2 \theta$
4.  $\cos \theta (\sec \theta - \cos \theta) = \sin^2 \theta$
5.  $\tan^2 \theta = (\sec \theta + 1)(\sec \theta - 1)$
6.  $\sin^2 \theta = (1 - \cos \theta)(1 + \cos \theta)$
7.  $(\csc \theta - 1)(\csc \theta + 1) = \cot^2 \theta$
8.  $(\sec \theta - 1)(\sec \theta + 1) = \tan^2 \theta$
9.  $\cos^2 \theta + \cos^2 \theta \tan^2 \theta = 1$
10.  $\csc^2 \theta + \csc^2 \theta \cot^2 \theta = \csc^4 \theta$
11.  $\sec^2 \theta \tan^2 \theta - \tan^2 \theta = \tan^4 \theta$
12.  $\sec^2 \theta \tan^2 \theta + \sec^2 \theta = \sec^4 \theta$
13.  $\sec^4 \theta - \tan^4 \theta = \sec^2 \theta + \tan^2 \theta$
14.  $\cot^4 \theta - \csc^4 \theta = -\cot^2 \theta - \csc^2 \theta$
15.  $2 \cos^4 \theta + \cos^2 \theta \sin^2 \theta - \sin^4 \theta = 2 \cos^2 \theta - \sin^2 \theta$
16.  $\sec^4 \theta - 2 \sec^2 \theta + 1 = \tan^4 \theta$
17.  $\csc^4 \theta - 2 \csc^2 \theta \cot^2 \theta + \cot^4 \theta = 1$
18.  $\frac{\sec^4 \theta - 1}{\tan^2 \theta} = \sec^2 \theta + 1$
19.  $\frac{1 - \sin^4 \theta}{1 + \sin^2 \theta} = \cos^2 \theta$
20.  $\frac{\csc^3 \theta - \csc \theta + \cot \theta}{\csc \theta} = \cot^2 \theta + \cos \theta$
21.  $\frac{\cos^3 \theta - \cos \theta + \sin \theta}{\cos \theta} = \tan \theta - \sin^2 \theta$
22.  $\frac{\sec \theta \csc^3 \theta - \csc \theta \sec \theta + 1}{\sec \theta} = \csc \theta \cot^2 \theta + \cos \theta$
23.  $\frac{\cot \theta - \cot \theta \cos^2 \theta + 1}{\sin \theta} = \cos \theta + \csc \theta$
24.  $\frac{\sin \theta - \sin^3 \theta + \cos \theta}{\sin \theta} = \cos^2 \theta + \cot \theta$
25.  $\frac{\sin \theta + \cos \theta \tan \theta}{\tan \theta} = 2 \cos \theta$
26.  $\frac{\cos \theta + \cos \theta \tan^2 \theta + 2 \sin \theta}{\cos \theta} = (1 + \tan \theta)^2$
27.  $\frac{\sec \theta}{\tan \theta} + \frac{\tan \theta}{\cos \theta} = \sec^2 \theta \csc \theta$
28.  $\frac{\sin \theta}{\cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = \frac{1 + \cos \theta}{\cos \theta \sin \theta}$
29.  $\frac{1 + \sec \theta}{\tan \theta} - \frac{\tan \theta}{\sec \theta} = \frac{\sec \theta + 1}{\tan \theta \sec \theta}$
30.  $\frac{\csc \theta}{\cot \theta} + \frac{1 - \cot \theta}{\csc \theta} = \frac{\cot \theta + 1}{\cot \theta \csc \theta}$
31.  $\frac{\csc \theta - \sin \theta}{\cot \theta} - \frac{\cot \theta}{\csc \theta} = 0$
32.  $\frac{\cos \theta}{1 - \sin \theta} - \frac{1 + \sin \theta}{\cos \theta} = 0$
33.  $\frac{1 - \sin \theta}{\cos \theta} + \frac{\cos \theta}{1 - \sin \theta} = 2 \sec \theta$
34.  $\frac{\cos \theta}{1 + \csc \theta} - \frac{\cos \theta}{1 - \csc \theta} = 2 \tan \theta$
35.  $\frac{\cot \theta}{\sec \theta - \tan \theta} - \frac{\cos \theta}{\sec \theta + \tan \theta} = \csc \theta + \sin \theta$
36.  $\frac{1 + \sin \theta}{\cos \theta} = \frac{\cos \theta}{1 - \sin \theta}$

37.  $\frac{1 - \cos \theta}{\sin^2 \theta} = \frac{1}{1 + \cos \theta}$
38.  $\frac{1 + \sin \theta}{\sin \theta} = \frac{\cos \theta \cot \theta}{1 - \sin \theta}$
39.  $\frac{1 + \cos \theta}{\tan^2 \theta} = \frac{\cos \theta}{\sec \theta - 1}$
40.  $\frac{\sin \theta}{1 - \cos \theta} + \frac{1}{1 + \cos \theta} = \frac{\sin \theta(1 + \cos \theta) + 1 - \cos \theta}{\sin^2 \theta}$
41.  $\frac{\tan \theta - \cot \theta}{1 - \cot \theta} = \tan \theta + 1$
42.  $\frac{\cos \theta \sin \theta}{\cos^2 \theta - \sin^2 \theta} = \frac{\tan \theta}{1 - \tan^2 \theta}$
43.  $\frac{\cos^2 \theta}{\csc \theta - 1} = \frac{\sin \theta + 1}{\csc \theta}$
44.  $\frac{\sin \theta + 1}{\csc \theta} = \frac{\cos^2 \theta}{\csc \theta - 1}$
45.  $\csc \theta = \cot \theta + \frac{1}{\csc \theta + \cot \theta}$
46.  $\sec \theta = \tan \theta + \frac{\cos \theta}{1 + \sin \theta}$
47.  $1 = \frac{\sin \theta \cot \theta}{\sec \theta - \tan \theta} - \sin \theta$
48.  $\sec \theta - \cos \theta = \frac{\cos \theta \sin \theta}{1 - \sin \theta} - \tan \theta$
49.  $\frac{2 + \sin \theta - 3 \sin^2 \theta}{\cos^2 \theta} = \frac{2 + 3 \sin \theta}{1 + \sin \theta}$
50.  $\frac{\sin^4 \theta - \sin^2 \theta \cos^2 \theta - 2 \cos^4 \theta}{1 - 3 \cos^2 \theta} = 1$
51.  $\frac{4 \cos^2 \theta - 5 \cos \theta + 1}{1 - \cos^2 \theta} = \frac{1 - 4 \cos \theta}{1 + \cos \theta}$
52.  $\frac{\sin^3 \theta + \cos^3 \theta}{2 \cos^2 \theta - 1} = \frac{\sec \theta - \sin \theta}{1 - \tan \theta}$
53.  $\frac{1 + \sin \theta}{1 - \sin \theta} - \frac{\sec \theta - \tan \theta}{\sec \theta + \tan \theta} = 4 \tan \theta \sec \theta$
54.  $\frac{\csc \theta - \cot \theta}{\csc \theta + \cot \theta} - \frac{1 + \cos \theta}{1 - \cos \theta} = \frac{4 \sec \theta}{1 - \sec^2 \theta}$
55.  $(5 \sin \theta - 4 \cos \theta)^2 + (4 \sin \theta + 5 \cos \theta)^2 = 41$
56.  $\frac{\csc^3 \theta - \sin^3 \theta}{\csc \theta - \sin \theta} = \csc^2 \theta + 1 + \sin^2 \theta$

### Digging Deeper

57.  $\frac{1 + \cos \theta + \sin \theta}{1 + \cos \theta - \sin \theta} = \frac{1 + \sin \theta}{\cos \theta}$  (Hint: Multiply the numerator and the denominator of the left side by  $1 + \cos \theta + \sin \theta$ )