Review and Preview to Chapter 5

EXERCISE 1

1. (a) $y = 25 - 4x^2$. When $x = 0$, $y = 25$. When $y = 0$, $4x^2 = 25$, so $x = \pm\frac{5}{2}$.

(b) $y = 2x^2 - x - 1$. When $x = 0$, $y = -1$. When $y = 0$, $2x^2 - x - 1 = 0$, $(2x+1)(x-1) = 0$, so $x = -\frac{1}{2}$, $x = 1$.

(c) $y = \dfrac{x^2 + 2x - 3}{x^2 + 1}$. When $x = 0$, $y = -3$. When $y = 0$, $x^2 + 2x - 3 = 0$, $(x-1)(x+3) = 0$, so $x = 1$, $x = -3$.

(d) $y = x^2 + x + 1$. When $x = 0$, $y = 1$. When $y = 0$, $x^2 + x + 1 = 0$, so $x = \dfrac{-1 \pm \sqrt{1-4}}{2}$. Therefore, there is no x-intercept.

(e) $y = 3x^2 + 4x - 6$. When $x = 0$, $y = -6$. When $y = 0$, $3x^2 + 4x - 6 = 0$, so $x = \dfrac{-4 \pm \sqrt{16+72}}{6} = \dfrac{-2 \pm \sqrt{22}}{3}$.

(f) $y = x^3 - 3x$. When $x = 0$, $y = 0$. When $y = 0$, $x^3 - 3x = 0$, $x(x^2 - 3) = 0$, so $x = 0$, $x = \pm\sqrt{3}$.

(g) $y = x^3 - x^2 - x + 1$. When $x = 0$, $y = 1$. When $y = 0$, $x^3 - x^2 - x + 1 = 0$, $x^2(x-1) - (x-1) = 0$, thus $(x+1)(x-1)(x-1) = 0$, so $x = -1$, $x = 1$.

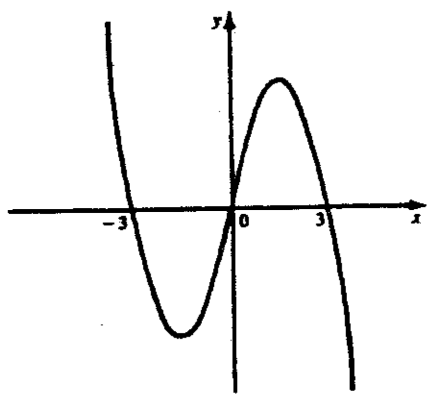
(h) $y = 2x^3 - 9x^2 - 18x$. When $x = 0$, $y = 0$. When $y = 0$, $2x^3 - 9x^2 - 18x = 0$, $x(2x^2 - 9x - 18) = 0$, thus $x(2x+3)(x-6) = 0$, so $x = 0$, $x = -\frac{3}{2}$, $x = 6$.

(i) $y = x^3 + 8$. When $x = 0$, $y = 8$. When $y = 0$, $x^3 = -8$, so $x = -2$.

(j) $y = x^4 - 16$. When $x = 0$, $y = -16$. When $y = 0$, $x^4 = 16$, so $x = \pm 2$.

2. $y = 9x - x^3$. When $x = 0$, $y = 0$. When $y = 0$, $9x - x^3 = 0$, $x(9 - x^2) = 0$, so $x = 0$, $x = \pm 3$. $y' = 9 - 3x^2$, so $y'(x) = 0$ at $\pm\sqrt{3}$.

Interval	$9 - 3x^2$	f
$(-\infty, -\sqrt{3})$	$-$	decreases
$(-\sqrt{3}, \sqrt{3})$	$+$	increases
$(\sqrt{3}, \infty)$	$-$	decreases



Therefore $f(-\sqrt{3}) = -6\sqrt{3}$ is a local minimum and $f(\sqrt{3}) = 6\sqrt{3}$ is a local maximum.

3. $f(x) = x^3 - 3x + 1$, $f'(x) = 3x^2 - 3$. $x_{n+1} = x_n - \frac{x_n^3 - 3x_n + 1}{3x_n^2 - 3}$.

There are three intercepts.

Guess: $x_1 = 2$, $x_2 = 1.667$, $x_3 = 1.549$, $x_4 = 1.532$, $x_5 = 1.53$.

Guess: $x_1 = 0$, $x_2 = 0.333$, $x_3 = 0.347$, $x_4 = 0.35$.

Guess: $x_1 = -2$, $x_2 = -1.889$, $x_3 = -1.879$, $x_4 = -1.88$.

EXERCISE 2

1. (a) odd. (b) even. (c) neither. (d) odd.

2. (a) $f(x) = x^2$, $f(-x) = x^2$, so the function is even.

(b) $f(x) = x^3$, $f(-x) = -x^3$, so the function is odd.

(c) $g(x) = x^2 + x^3$, $g(-x) = x^2 - x^3$, so the function is neither.

(d) $g(x) = \frac{2}{x^4+1}$, $g(-x) = \frac{2}{x^4+1}$, so the function is even.

(e) $h(x) = (x + x^5)^3$, $h(-x) = (-x - x^5)^3 = -(x + x^5)^3$, so the function is odd.

(f) $h(x) = x^6(1 + x - x^2)$, $h(-x) = x^6(1 - x - x^2)$, so the function is neither.

(g) $y = |x|$, $f(-x) = |x|$, so the function is even.

(h) $y = \frac{x^3}{x^4 + x^2 + 1}$, $f(-x) = -\frac{x^3}{x^4 + x^2 + 1}$, so the function is odd.

Exercise 5.1

1. (a) The vertical asymptotes are: $x = -7$, $x = -3$, $x = 2$, and $x = 6$.

(b) (i) $\lim_{x \to -7^-} f(x) = -\infty$. (ii) $\lim_{x \to -7^+} f(x) = \infty$.

(iii) $\lim_{x \to -3} f(x) = \infty$. (iv) $\lim_{x \to 2} f(x) = -\infty$.

(v) $\lim_{x \to 6^-} f(x) = \infty$. (vi) $\lim_{x \to 6^+} f(x) = -\infty$.

2. (a) $\lim_{x \to 8} \frac{1}{(x-8)^2} = \infty$. (b) $\lim_{x \to 1^-} \frac{3}{x-1} = -\infty$.

(c) $\lim_{x \to 1^+} \frac{3}{x-1} = \infty$. (d) $\lim_{x \to -1} \frac{-2}{(x+1)^2} = -\infty$.

(e) $\lim_{x \to 2^+} \frac{x-4}{x-2} = -\infty$. (f) $\lim_{x \to 2^-} \frac{x-4}{x-2} = \infty$.

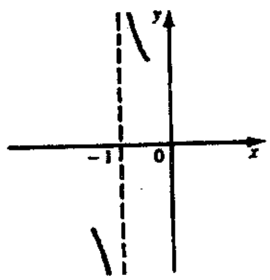
(g) $\lim_{x \to -4}\left[1 + \frac{2x}{(x+4)^6}\right] = -\infty$. (h) $\lim_{x \to 3^+}\left[x + \frac{2-x}{x-3}\right] = -\infty$.

(i) $\lim_{x \to -2^+} \frac{x}{x^2-4} = \infty$. (j) $\lim_{x \to -2^-} \frac{x}{x^2-4} = -\infty$.

(k) $\lim_{x \to 9^+} \frac{5-x}{\sqrt{x-9}} = -\infty$. (l) $\lim_{x \to -3^+} \frac{10}{x^2-x-12} = \lim_{x \to -3^+} \frac{10}{(x+3)(x-4)} = -\infty$.

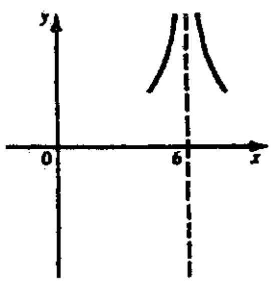
3. (a) $y = \frac{2}{x+1}$, $\lim_{x \to -1^-} y = -\infty$, and $\lim_{x \to -1^+} y = \infty$.

Therefore, $x = -1$ is a vertical asymptote.



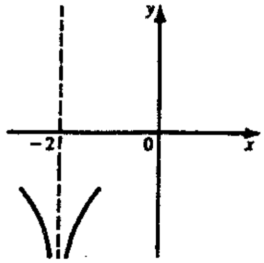
(b) $y = \frac{3}{(x-6)^2}$, $\lim_{x \to 6} y = \infty$.

Therefore $x = 6$ is a vertical asymptote.



(c) $y = \frac{x}{(x+2)^2}$, $\lim_{x \to -2} y = -\infty$.

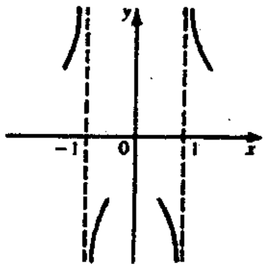
Therefore $x = -2$ is a vertical asymptote.



(d) $y = \frac{1}{x^2 - 1}$, $\lim_{x \to -1^-} y = \infty$ and $\lim_{x \to -1^+} y = -\infty$.

$\lim_{x \to 1^-} y = -\infty$ and $\lim_{x \to 1^+} y = \infty$. Therefore,

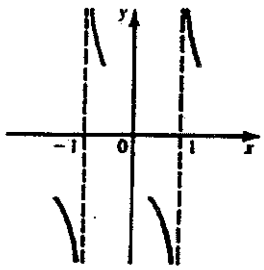
$x = -1$ and $x = 1$ are vertical asymptotes.



(e) $y = \frac{x}{x^2 - 1}$, $\lim_{x \to -1^-} y = -\infty$ and $\lim_{x \to -1^+} y = \infty$.

$\lim_{x \to 1^-} y = -\infty$ and $\lim_{x \to 1^+} y = \infty$. Therefore,

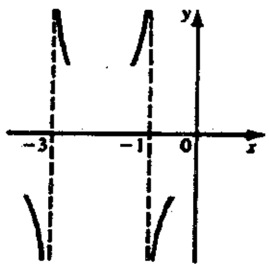
$x = -1$ and $x = 1$ are vertical asymptotes.



(f) $y = \frac{6x^3}{x^2 + 4x + 3}$, $\lim_{x \to -1^-} y = \infty$ and $\lim_{x \to -1^+} y = -\infty$.

$\lim_{x \to -3^-} y = -\infty$ and $\lim_{x \to -3^+} y = \infty$. Therefore,

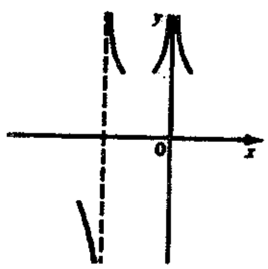
$x = -3$ and $x = -1$ are vertical asymptotes.



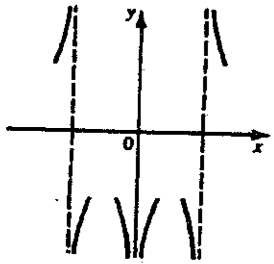
(g) $y = \frac{1}{x^2(x+1)}$, $\lim_{x \to 0} y = \infty$.

$\lim_{x \to -1^-} y = -\infty$ and $\lim_{x \to -1^+} y = \infty$. Therefore,

$x = -1$ and $x = 0$ are vertical asymptotes.



(h) $y = \frac{1}{x^4 - 4x^2} = \frac{1}{x^2(x+2)(x-2)}$, $\lim_{x \to 0} y = -\infty$.



$\lim_{x \to -2^-} y = \infty$ and $\lim_{x \to -2^+} y = -\infty$.

$\lim_{x \to 2^-} y = -\infty$ and $\lim_{x \to 2^+} y = \infty$. Therefore,
$x = -2$, $x = 0$, and $x = 2$ are vertical asymptotes.

4. $\lim_{x \to 0^+} \left[\frac{5}{x} - \frac{2}{x^2}\right] = \lim_{x \to 0^+} \left[\frac{5x-2}{x^2}\right] = -\infty$.

5. $\frac{1}{x^4} > 100\,000\,000 = 10^8$. So $x^4 < \frac{1}{10^8}$, then $|x| < \frac{1}{10^2}$.

Exercise 5.2

1. (a) Horizontal asymptote: $y = 2$. Vertical asymptotes: $x = -2$ and $x = 1$.
(b) Horizontal asymptotes: $y = -2$ and $y = 1$. Vertical asymptotes: $x = -1$ and $x = 4$.

2. (a) $\lim_{x\to\infty} \frac{6}{\sqrt{x}} = 0.$ (b) $\lim_{x\to-\infty} 3x^{-6} = 0.$

(c) $\lim_{x\to\infty} \frac{2x+1}{x-3} = \lim_{x\to\infty} \frac{2+\frac{1}{x}}{1-\frac{3}{x}} = \frac{2+0}{1-0} = 2.$

(d) $\lim_{x\to-\infty} \frac{2x+1}{x-3} = \lim_{x\to-\infty} \frac{2+\frac{1}{x}}{1-\frac{3}{x}} = \frac{2+0}{1-0} = 2.$

(e) $\lim_{x\to\infty} \frac{1-x}{3+5x} = \lim_{x\to\infty} \frac{\frac{1}{x}-1}{\frac{3}{x}+5} = \frac{0-1}{0+5} = -\frac{1}{5}$

(f) $\lim_{x\to-\infty} \frac{x^2-x+1}{x^2+3x-2} = \lim_{x\to-\infty} \frac{1-\frac{1}{x}+\frac{1}{x^2}}{1+\frac{3}{x}-\frac{2}{x^2}} = \frac{1-0+0}{1+0-0} = 1.$

(g) $\lim_{x\to\infty} \frac{x+3}{x^2-5x+7} = \lim_{x\to\infty} \frac{\frac{1}{x}+\frac{3}{x^2}}{1-\frac{5}{x}+\frac{7}{x^2}} = \frac{0+0}{1-0+0} = 0.$

(h) $\lim_{x\to\infty} \frac{x^2-1}{(x+3)(2x+4)} = \lim_{x\to\infty} \frac{1-\frac{1}{x^2}}{2+\frac{10}{x}+\frac{12}{x^2}} = \frac{1-0}{2+0+0} = \frac{1}{2}.$

(i) $\lim_{x\to-\infty} \frac{3x^3+x^2-5}{x^3-4x+1} = \lim_{x\to-\infty} \frac{3+\frac{1}{x}-\frac{5}{x^3}}{1-\frac{4}{x^2}+\frac{1}{x^3}} = \frac{3+0-0}{1-0+0} = 3.$

(j) $\lim_{x\to-\infty} \frac{12x^2-2x+1}{3x^4-14x^2+x-3} = \lim_{x\to-\infty} \frac{\frac{12}{x^2}-\frac{2}{x^3}+\frac{1}{x^4}}{3-\frac{14}{x^2}+\frac{1}{x^3}-\frac{3}{x^4}}$

$= \frac{0-0+0}{3-0+0-0} = 0.$

3. (a) $\lim_{x\to\pm\infty} \frac{2x-3}{5-4x} = \lim_{x\to\pm\infty} \frac{2-\frac{3}{x}}{\frac{5}{x}-4} = \frac{2-0}{0-4} = -\frac{1}{2}.$ Therefore $y = -\frac{1}{2}$ is a horizontal asymptote.

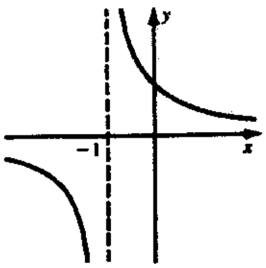
(b) $\lim\limits_{x \to \pm\infty} \frac{x}{x^2+1} = \lim\limits_{x \to \pm\infty} \frac{\frac{1}{x}}{1+\frac{1}{x^2}} = \frac{0}{1+0} = 0$. Therefore $y = 0$ is a horizontal asymptote.

(c) $\lim\limits_{x \to \pm\infty} \frac{x^3+1}{x^3-1} = \lim\limits_{x \to \pm\infty} \frac{1+\frac{1}{x^3}}{1-\frac{1}{x^3}} = \frac{1+0}{1-0} = 1$. Therefore $y = 1$ is a horizontal asymptote.

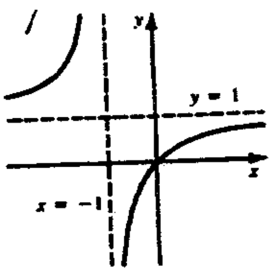
(d) $\lim\limits_{x \to \pm\infty} \left[1 - \frac{x}{x^2-2}\right] = \lim\limits_{x \to \pm\infty} \frac{x^2-2-x}{x^2-2} = \lim\limits_{x \to \pm\infty} \frac{1-\frac{2}{x^2}-\frac{1}{x}}{1-\frac{2}{x^2}} = \frac{1-0-0}{1-0} = 1$.

Therefore $y = 1$ is a horizontal asymptote.

4. (a) $y = \frac{2}{x+1}$. $\lim\limits_{x \to -1^-} \frac{2}{x+1} = -\infty$ and $\lim\limits_{x \to -1^+} \frac{2}{x+1} = \infty$, so there is a vertical asymptote at $x = -1$. $\lim\limits_{x \to \pm\infty} \frac{2}{x+1} = \lim\limits_{x \to \pm\infty} \frac{\frac{2}{x}}{1+\frac{1}{x}} = 0$, so there is a horizontal asymptote at $y = 0$. There is no x-intercept and the y-intercept is 2.

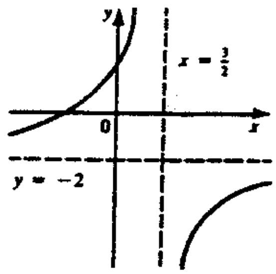


(b) $y = \frac{x}{x+1}$. $\lim\limits_{x \to -1^-} \frac{x}{x+1} = \infty$ and $\lim\limits_{x \to -1^+} \frac{x}{x+1} = -\infty$, so there is a vertical asymptote at $x = -1$. $\lim\limits_{x \to \pm\infty} \frac{x}{x+1} = \lim\limits_{x \to \pm\infty} \frac{1}{1+\frac{1}{x}} = 1$, so there is a horizontal asymptote at $y = 1$. The x-intercept is 0 and the y-intercept is 0.



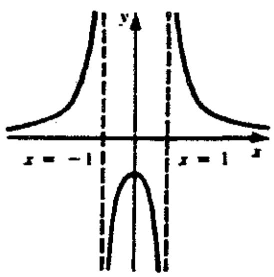
Exercise 5.2

(c) $y = \frac{4x+5}{3-2x}$. $\lim_{x \to \frac{3}{2}^-} \frac{4x+5}{3-2x} = \infty$ and $\lim_{x \to \frac{3}{2}^+} \frac{4x+5}{3-2x} = -\infty$, so there is a vertical asymptote at $x = \frac{3}{2}$. $\lim_{x \to \pm\infty} \frac{4x+5}{3-2x} = \lim_{x \to \pm\infty} \frac{4+\frac{5}{x}}{\frac{3}{x}-2} = -2$, so there is a horizontal asymptote at $y = -2$. The x-intercept is $-\frac{5}{4}$ and the y-intercept is $\frac{5}{3}$.



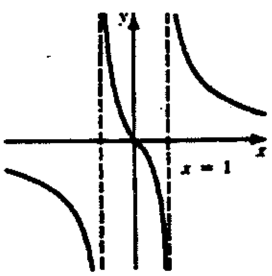
(d) $y = \frac{1}{x^2-1}$. $\lim_{x \to -1^-} \frac{1}{x^2-1} = \infty$ and $\lim_{x \to -1^+} \frac{1}{x^2-1} = -\infty$.

$\lim_{x \to 1^-} \frac{1}{x^2-1} = -\infty$ and $\lim_{x \to 1^+} \frac{1}{x^2-1} = \infty$. So there are vertical asymptotes at $x = -1$ and $x = 1$. $\lim_{x \to \pm\infty} \frac{1}{x^2-1} = \lim_{x \to \pm\infty} \frac{\frac{1}{x^2}}{1-\frac{1}{x^2}} = 0$, so there is a horizontal asymptote at $y = 0$. There is no x-intercept and the y-intercept is -1.



(e) $y = \frac{x}{x^2-1}$. $\lim_{x \to -1^-} \frac{x}{x^2-1} = -\infty$ and $\lim_{x \to -1^+} \frac{x}{x^2-1} = \infty$.

$\lim_{x \to 1^-} \frac{x}{x^2-1} = -\infty$ and $\lim_{x \to 1^+} \frac{x}{x^2-1} = \infty$. So there are vertical asymptotes at $x = -1$ and $x = 1$. $\lim_{x \to \pm\infty} \frac{x}{x^2-1} = \lim_{x \to \pm\infty} \frac{\frac{1}{x}}{1-\frac{1}{x^2}} = 0$, so there is a horizontal asymptote at $y = 0$. The x-intercept is 0 and the y-intercept is 0.

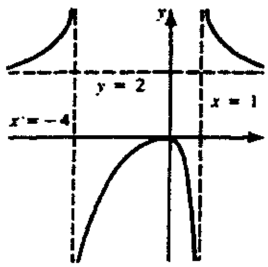


(f) $y = \frac{2x^2}{x^2+3x-4} = \frac{2x^2}{(x+4)(x-1)}$. $\lim_{x\to -4^-} y = \infty$ and $\lim_{x\to -4^+} y = -\infty$.

$\lim_{x\to 1^-} y = -\infty$ and $\lim_{x\to 1^+} y = \infty$. So there are vertical asymptotes at $x = -4$

and $x = 1$. $\lim_{x\to\pm\infty} \frac{2x^2}{x^2+3x-4} = \lim_{x\to\pm\infty} \frac{2}{1+\frac{3}{x}-\frac{4}{x^2}} = 2$, so there is a horizontal

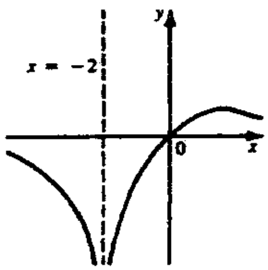
asymptote at $y = 2$. The x-intercept is 0 and the y-intercept is 0.



(g) $y = \frac{x}{(x+2)^2}$. $\lim_{x\to -2} \frac{x}{(x+2)^2} = -\infty$, so there is a vertical asymptote at

$x = -2$. $\lim_{x\to\pm\infty} \frac{x}{(x+2)^2} = \lim_{x\to\pm\infty} \frac{\frac{1}{x}}{1+\frac{4}{x}+\frac{4}{x^2}} = 0$, so there is a horizontal

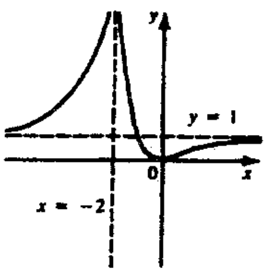
asymptote at $y = 0$. The x-intercept is 0 and the y-intercept is 0.



(h) $y = \frac{x^2}{(x+2)^2}$. $\lim_{x\to -2} \frac{x^2}{(x+2)^2} = \infty$, so there is a vertical asymptote at

$x = -2$. $\lim_{x\to\pm\infty} \frac{x^2}{(x+2)^2} = \lim_{x\to\pm\infty} \frac{1}{1+\frac{4}{x}+\frac{4}{x^2}} = 1$, so there is a horizontal

asymptote at $y = 1$. The x-intercept is 0 and the y-intercept is 0.



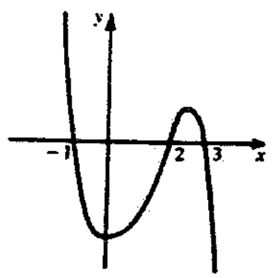
5. (a) $\lim_{x \to \infty} \sqrt{x} = \infty$. (b) $\lim_{x \to -\infty} x^6 = -\infty$.

(c) $\lim_{x \to \infty} (x^3 - x^2) = \lim_{x \to \infty} x^2(x-1) = \infty$ since $x^2 \to \infty$ and $x - 1 \to \infty$.

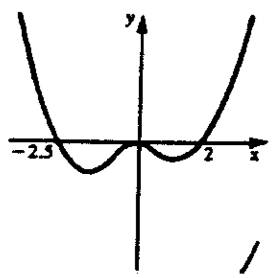
(d) $\lim_{x \to -\infty} (x^3 - x^2) = \lim_{x \to -\infty} x^2(x-1) = -\infty$ since $x^2 \to \infty$ and $x - 1 \to -\infty$.

(e) $\lim_{x \to \infty} x^2(2x+1)(x-2) = \infty$. (f) $\lim_{x \to \infty} (x+2)^4(3-x) = -\infty$.

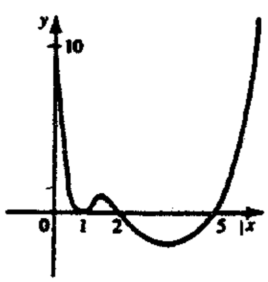
6. (a) $\lim_{x \to \infty} (x+1)(x-2)(3-x) = -\infty$ and $\lim_{x \to -\infty} (x+1)(x-2)(3-x) = \infty$.
The x-intercepts are -1, 2, and 3, and the y-intercept is -6.



(b) $\lim_{x \to \infty} x^2(x-2)(2x+5) = \infty$ and $\lim_{x \to -\infty} x^2(x-2)(2x+5) = \infty$.
The x-intercepts are 0, 2, and $-\frac{5}{2}$, and the y-intercept is 0.

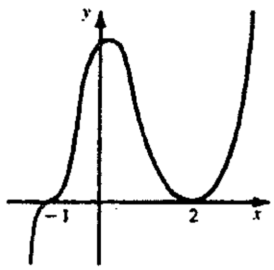


(c) $\lim_{x \to \infty} (1-x)^2(2-x)(5-x) = \infty$ and $\lim_{x \to -\infty} (1-x)^2(2-x)(5-x) = \infty$.
The x-intercepts are 1, 2, and 5, and the y-intercept is 10.



(d) $\lim_{x\to\infty}(x+1)^3(x-2)^4 = \infty$ and $\lim_{x\to-\infty}(x+1)^3(x-2)^4 = -\infty$.

The x-intercepts are -1 and 2, and the y-intercept is 16.



7. $\lim_{x\to\infty}\frac{x}{|x|+1} = \lim_{x\to\infty}\frac{x}{x+1} = \lim_{x\to\infty}\frac{1}{1+\frac{1}{x}} = \frac{1}{1+0} = 1$

$|x| = -x$ for $x<0$, so $\lim_{x\to-\infty}\frac{x}{|x|+1} = \lim_{x\to-\infty}\frac{x}{-x+1} = \lim_{x\to\infty}\frac{1}{-1+\frac{1}{x}} = -1$,

so the horizontal asymptotes are $y = 1$ and $y = -1$.

8. Divide num. and denom. by x: $\lim_{x\to\infty}\frac{\sqrt{4x^2+1}}{2x-3} = \lim_{x\to\infty}\frac{\sqrt{4+\frac{1}{x^2}}}{2-\frac{3}{x}} = \frac{\sqrt{4+0}}{2-0} = 1$.

9. $\lim_{x\to-\infty}\frac{3x}{\sqrt{x^2+6}} = \lim_{x\to-\infty}\frac{3}{-\sqrt{1+\frac{6}{x^2}}} = -3$ (since $\sqrt{x^2} = -x$ for $x < 0$).

10. Rationalize the numerator: $\lim_{x\to\infty}(\sqrt{x^2+5x+1}-x) = \lim_{x\to\infty}\frac{x^2+5x+1-x^2}{\sqrt{x^2+5x+1}+x}$

$= \lim_{x\to\infty}\frac{5+\frac{1}{x}}{\sqrt{1+\frac{5}{x}+\frac{1}{x^2}}+1} = \frac{5+0}{\sqrt{1+0+0}+1} = \frac{5}{2}$.

11. $\lim_{x\to-\infty}\frac{x^{10}+6x^6-3}{x^5+2x} = \lim_{x\to-\infty}\frac{x^5+6x-\frac{3}{x^5}}{1+\frac{2}{x^4}} = -\infty$

since numerator $\to -\infty$ and denominator $\to 1$.

12. $\frac{1}{x^2}<0.000001$, so $x^2>\frac{1}{0.000001} = 10^6$, thus $x>10^3$. So we have to take x larger than 1000.

Exercise 5.3

1. (a) CU on $(-6,-3)$, $(-1,1)$, $(1,3)$ and $(10,13)$.

CD on $(-10,-6)$, $(-3,-1)$, $(3,7)$ and $(7,10)$.

(b) The points of inflection are: $(-6,-1)$, $(-3,0)$, $(-1,2)$, $(3,1)$, and $(10,2)$.

2. (a) $y = 2+5x-12x^2$, $y' = 5-24x$, $y'' = -24<0$, so CD on $(-\infty,\infty)$.

(b) $y = 6x^2-12x+1$, $y' = 12x-12$, $y'' = 12>0$, so CU on $(-\infty,\infty)$.

(c) $y = 16+4x+x^2-x^3$, $y' = 4+2x-3x^2$, $y'' = 2-6x$.

When $2-6x>0$, $x<\frac{1}{3}$, so CU on $(-\infty,\frac{1}{3})$. When $2-6x<0$, $x>\frac{1}{3}$, so CD on $(\frac{1}{3},\infty)$.

The inflection point is $\left(\frac{1}{3},\frac{470}{27}\right)$.

(d) $y = 2x^3+24x^2-5x-21$, $y' = 6x^2+48x-5$, $y'' = 12x+48$.

When $12x+48>0$, $x>-4$, so CU on $(-4,\infty)$. When $12x+48<0$, $x<-4$, so CD on $(-\infty,-4)$. The inflection point is $(-4,255)$.

(e) $y = x^4-2x^3+x-2$, $y' = 4x^3-6x^2+1$, $y'' = 12x^2-12x = 12x(x-1)$.

When $12x(x-1)>0$, $x>0$ and $x>1$, or, $x<0$ and $x<1$, so CU on $(-\infty,0)$ and $(1,\infty)$.

When $12x(x-1)<0$, $x<0$ and $x>1$, which is impossible, or, $x>0$ and $x<1$, so CD on $(0,1)$. The inflection points are $(0,-2)$ and $(1,-2)$.

(f) $y = x^4-24x^2+x-1$, $y' = 4x^3-48x+1$, $y'' = 12x^2+48$.

When $12x^2-48>0$, $|x|>2$, so CU on $(-\infty,-2)$ and $(2,\infty)$. When $12x^2-48<0$, $x^2<4 \Rightarrow |x|<2$, so CD on $(-2,2)$. The inflection points are $(-2,-83)$ and $(2,-79)$.

(g) $y = \frac{1}{x-1}$, $y' = -\frac{1}{(x-1)^2}$, $y'' = \frac{2x-2}{(x-1)^4} = \frac{2}{(x-1)^3}$.

When $x-1>0$, $x>1$, so CU on $(1,\infty)$. When $x-1<0$, $x<1$, so CD on $(-\infty,1)$.

There is no point of inflection since y does not exist when $x = 1$.

(h) $y = \frac{x-2}{5-x}$, $y' = \frac{(5-x)+(x-2)}{(5-x)^2} = \frac{3}{(5-x)^2}$, $y'' = \frac{30-6x}{(5-x)^4} = \frac{6}{(5-x)^3}$.

When $5-x>0$, $x<5$, so CU on $(-\infty,5)$. When $5-x<0$, $x>5$, so CD on $(5,\infty)$.

There is no point of inflection since y does not exist when $x = 5$.

(i) $y = \frac{1}{x^2+1}$, $y' = \frac{-2x}{(x^2+1)^2}$, $y'' = \frac{-2(x^2+1)^2+8x^2(x^2+1)}{(x^2+1)^4} = \frac{2(-x^2-1+4x^2)}{(x^2+1)^3}$

$= \frac{2(3x^2-1)}{(x^2+1)^3}$. Now $(x^2+1)^3$ is always positive. When $3x^2-1>0$, $x^2>\frac{1}{3} \Rightarrow |x|>\frac{1}{\sqrt{3}}$,

so CU on $(-\infty,-\frac{1}{\sqrt{3}})$ and $(\frac{1}{\sqrt{3}},\infty)$. When $3x^2-1<0$, $|x|<\frac{1}{\sqrt{3}}$, so CD on $(-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}})$.

The inflection points are $(-\frac{1}{\sqrt{3}},\frac{3}{4})$ and $(\frac{1}{\sqrt{3}},\frac{3}{4})$.

(j) $y = \frac{1-x^2}{x^3}$, $y' = \frac{-2x^4-3x^2+3x^4}{x^6} = \frac{x^2-3}{x^4}$,

$y'' = \frac{2x^5-4x^5+12x^3}{x^8} = \frac{12-2x^2}{x^6} = \frac{2(6-x^2)}{x^6}$.

Interval	$6-x^2$	x^5	y''	y
$(-\infty,-\sqrt{6})$	−	−	+	CU
$(-\sqrt{6},0)$	+	−	−	CD
$(0,\sqrt{6})$	+	+	+	CU
$(\sqrt{6},\infty)$	−	+	−	CD

The only points of inflection are $\left(-\sqrt{6},\frac{5}{(\sqrt{6})^3}\right)$ and $\left(\sqrt{6},-\frac{5}{(\sqrt{6})^3}\right)$.

(k) $y = x^{\frac{2}{3}}(5+x)$, $y' = \frac{10}{3}x^{-\frac{1}{3}}+\frac{5}{3}x^{\frac{2}{3}}$, $y'' = -\frac{10}{9}x^{-\frac{4}{3}}+\frac{10}{9}x^{-\frac{1}{3}} = \frac{10}{9}x^{-\frac{4}{3}}(x-1)$.
When $x-1>0$, $x>1$, so CU on $(1,\infty)$. When $x-1<0$, $x<1$, so CD on $(-\infty,1)$.
The inflection point is (1,6).

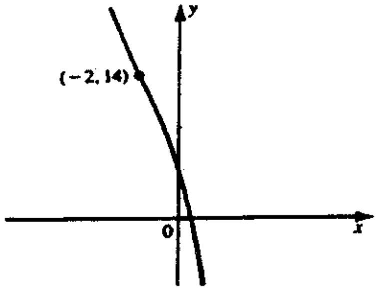
(l) $y = \frac{x^2}{\sqrt{x+1}}$, $y' = \frac{2x\sqrt{x+1}-\frac{x^2}{2\sqrt{x+1}}}{x+1} = \frac{\frac{x}{2\sqrt{x+1}}[4(x+1)-x]}{x+1} = \frac{3x^2+4x}{2(x+1)^{\frac{3}{2}}}$,

$y'' = \frac{(12x+8)(x+1)^{\frac{3}{2}}-3(3x^2+4x)\sqrt{x+1}}{4(x+1)^3} = \frac{12x^2+20x+8-9x^2-12x}{4(x+1)^{\frac{5}{2}}}$

$= \frac{3x^2+8x+8}{4(x+1)^{\frac{5}{2}}}$. Now $3x^2+8x+8>0$, for all x. When $x+1>0$, $x>-1$, so CU on $(-1,\infty)$. There are no inflection points. [Note: domain of y is $(-1,\infty)$.]

3. (i) (a) $y' = -13-12x-3x^2<0$, for all x. So y is decreasing on $(-\infty,\infty)$.
(b) There are no maximum or minimum values.
(c) $y'' = -12-6x$. When $-12-6x>0$, $x<-2$, so CU on $(-\infty,-2)$.
When $-12-6x<0$, $x>-2$, so CD on $(-2,\infty)$.
(d) The point of inflection is $(-2,14)$.

(e)



(ii) (a) $y' = 4x^3 - 16x = 4x(x^2 - 4)$.

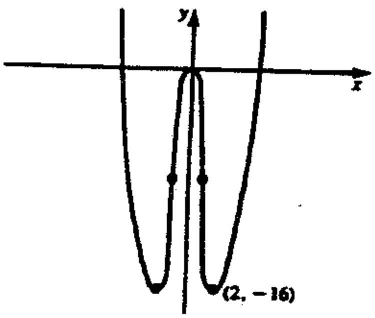
Interval	x	$x^2 - 4$	y''	y
$(-\infty, -2)$	−	+	−	decreasing
$(-2,0)$	−	−	+	increasing
$(0,2)$	+	−	−	decreasing
$(2,\infty)$	+	+	+	increasing

(b) $f(\pm 2) = -16$ are local minimum values and $f(0) = 0$ is a local maximum.

(c) $y'' = 12x^2 - 16$. When $12x^2 - 16 > 0$, $x^2 > \frac{4}{3} \Rightarrow |x| > \frac{2}{\sqrt{3}}$, so CU on $(-\infty, -\frac{2}{\sqrt{3}})$ and $(\frac{2}{\sqrt{3}},\infty)$. When $12x^2 - 16 < 0$, $|x| < \frac{2}{\sqrt{3}}$, so CD on $(-\frac{2}{\sqrt{3}},\frac{2}{\sqrt{3}})$.

(d) The points of inflection are $(-\frac{2}{\sqrt{3}}, -\frac{80}{9})$ and $(\frac{2}{\sqrt{3}}, -\frac{80}{9})$.

(e) ,



(iii) (a) $y' = \sqrt{x^2+4} + \frac{x^2}{\sqrt{x^2+4}} = \frac{2x^2+4}{\sqrt{x^2+4}} > 0$ for all x, so y increases on $(-\infty,\infty)$.

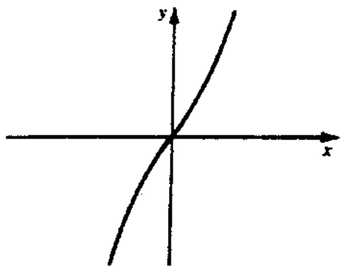
(b) There are no maximum or minimum values.

(c) $$y'' = \frac{4x\sqrt{x^2+4} - \frac{x(2x^2+4)}{\sqrt{x^2+4}}}{x^2+4} = \frac{4x^3 + 16x - 2x^3 - 4x}{(x^2+4)^{\frac{3}{2}}} = \frac{2x(x^2+6)}{(x^2+4)^{\frac{3}{2}}}.$$

Now (x^2+6) and (x^2+4) are always positive. So CU on $(0,\infty)$ and CD on $(-\infty,0)$.

(d) The point of inflection is (0,0).

(e)



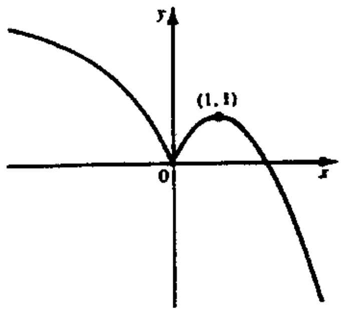
(iv) (a) $y' = 2x^{-\frac{1}{3}} - 2$. When $2x^{-\frac{1}{3}} - 2 > 0$, $0 < x < 1$, so y is increasing on (0,1). When $2x^{-\frac{1}{3}} - 2 < 0$, $x < 0$, or $x > 1$, so y is decreasing on $(-\infty,0)$ and $(1,\infty)$.

(b) $f(0) = 0$ is a minimum and $f(1) = 1$ is a maximum.

(c) $y'' = -\frac{2}{3}x^{-\frac{4}{3}}$. So CD on $(-\infty,0)$ and $(0,\infty)$.

(d) There are no inflection points.

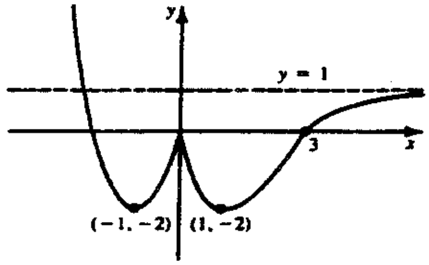
(e)



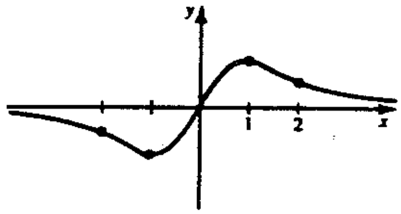
4. $y = x^3 + cx^2 + x + d$, $y' = 3x^2 + 2cx + 1$, $y'' = 6x + 2c$. When $x = 4$, $y'' = 0$, $6(4) + 2c = 0$, thus $c = -12$. So $y = x^3 - 12x^2 + x + d$. When $x = 4$ and $y = -7$, $-7 = 4^3 - 12(4^2) + 4 + d$, thus $d = 117$.

5. $f(x) = x|x|$. Thus $f(x) = x^2$ if $x \geq 0$ and $f(x) = -x^2$ if $x < 0$.
So $f''(x) = 2$ if $x > 0$ and $f''(x) = -2$ if $x < 0$ and $f''(0)$ does not exist.
Therefore $f''(x) > 0$ for $x > 0$ so f is CU on $(0,\infty)$ and when $x < 0$, $f''(x) < 0$, so CD on $(-\infty,0)$. The curve changes from CD to CU at 0, so there is an inflection point at (0,0), but $f''(0)$ does not exist.

6.



7.



8. $y = x^5 + 2x^3 + 6x^2 - 5x + 4$, $y' = 5x^4 + 6x^2 + 12x - 5$, $y'' = 20x^3 + 12x + 12$.
$x_{n+1} = x_n - \dfrac{20x_n^3 + 12x_n + 12}{60x_n^2 + 12}$. Now $x_1 = -1$, $x_2 \doteq -0.7222$, $x_3 \doteq -0.6252$, $x_4 \doteq -0.6362$, $x_5 \doteq -0.6142$, $x_6 \doteq -0.6141$. Thus, correct to three decimal places, the inflection point is $(-0.614, 8.782)$.

9. $g(x) = [f(x)]^2$, $g'(x) = 2f(x)f'(x)$, $g''(x) = 2f'(x)f'(x) + 2f(x)f''(x)$
$= 2\left[[f'(x)]^2 + f(x)f''(x)\right] > 0$ on I since $[f'(x)]^2 \geq 0$, $f(x) > 0$ on R, and $f''(x) > 0$ because f is CU on I. So g is CU on I.

Exercise 5.4

1. (a) $f(x) = 3x^2 - 4x + 13$, $f'(x) = 6x - 4 = 0$, so $x = \frac{2}{3}$. $f''(x) = 6 > 0$, thus $f(\frac{2}{3}) = \frac{35}{3}$ is a local minimum.

(b) $f(x) = 2 + 6x - 6x^2$, $f'(x) = 6 - 12x = 0$, so $x = \frac{1}{2}$. $f''(x) = -12 < 0$, thus $f(\frac{1}{2}) = \frac{7}{2}$ is a local maximum.

(c) $g(x) = 2x^3 - 48x - 17$, $g'(x) = 6x^2 - 48 = 0$, so $x = \pm 2\sqrt{2}$. $g''(x) = 12x$, $g''(2\sqrt{2}) > 0$, so $g(2\sqrt{2}) = -64\sqrt{2} - 17$ is a local minimum. $g''(-2\sqrt{2}) < 0$, so $g(-2\sqrt{2}) = 64\sqrt{2} - 17$ is a local maximum.

(d) $g(x) = 1 + 3x^2 - 2x^3$, $g'(x) = 6x - 6x^2 = 6x(1-x) = 0$, so $x = 0$, $x = 1$. $g''(x) = 6 - 12x$, $g''(0) > 0$, so $g(0) = 1$ is a local minimum. $g''(1) < 0$, so $g(1) = 2$ is a local maximum.

(e) $h(x) = x^3 - 9x^2 + 24x - 10$, $h'(x) = 3x^2 - 18x + 24 = 3(x-4)(x-2) = 0$, so $x = 4$, $x = 2$. $h''(x) = 6x - 18$, $h''(2) < 0$, so $h(2) = 10$ is a local maximum. $h''(4) > 0$, so $h(4) = 6$ is a local minimum.

(f) $h(x) = x^4 - x^3$, $h'(x) = 4x^3 - 3x^2 = x^2(4x - 3) = 0$, so $x = 0$, $x = \frac{3}{4}$. $h''(x) = 12x^2 - 6x$, $h''(0) = 0$, so there is no information on $h(0)$ from the second derivative test. $h''(\frac{3}{4}) > 0$, so $h(\frac{3}{4}) = -\frac{27}{256}$ is a local minimum.

(g) $F(x) = 3x^4 - 16x^3 + 18x^2 + 1$, $F'(x) = 12x^3 - 48x^2 + 36x = 12x(x^2 - 4x + 3)$ $= 12x(x-1)(x-3) = 0$, so $x = 0$, $x = 1$, $x = 3$. $F''(x) = 36x^2 - 96x + 36$, $F''(0) > 0$, so $F(0) = 1$ is a local minimum. $F''(1) < 0$, so $F(1) = 6$ is a local maximum. $F''(3) > 0$, so $F(3) = -26$ is a local minimum.

(h) $F(x) = 2 + 5x - x^5$, $F'(x) = 5 - 5x^4 = 0$, so $x = \pm 1$. $F''(x) = -20x^3$, $F''(-1) > 0$, so $F(-1) = -2$ is a local minimum. $F''(1) < 0$, so $F(1) = 6$ is a local maximum.

(i) $G(x) = (1 - 3x^2 + x^3)^5$, $G'(x) = 5(1 - 3x^2 + x^3)^4(-6x + 3x^2) = 0$, so $x = 0$, $x = 2$, and there are three other roots which can be disregarded since they will make $G''(x) = 0$.

$G''(x) = 5\left[4(1 - 3x^2 + x^3)^3(-6x + 3x^2)(-6x + 3x^2) + (1 - 3x^2 + x^3)^4(-6 + 6x)\right]$
$= 5(1 - 3x^2 + x^3)^3(42x^4 - 168x^3 + 162x^2 + 6x - 6)$. $G''(0) < 0$, so $G(0) = 1$ is a local maximum. $G''(2) > 0$, so $G(2) = -243$ is a local minimum.

(j) $G(x) = x^2 + \frac{16}{x}$, $G'(x) = 2x - \frac{16}{x^2} = 2x^{-2}(x^3 - 8) = 0$, so $x = 2$.
$G''(x) = 2 + \frac{32}{x^3}$, $G''(2) > 0$, so $G(2) = 12$ is a local minimum.

2. (a) $f(x) = x^4 - 6x^2 + 10$, $f'(x) = 4x^3 - 12x = 4x(x^2 - 3) = 0$, so $x = 0$, $x = \pm\sqrt{3}$. $f''(x) = 12x^2 - 12 = 12(x^2 - 1)$, $f''(0) < 0$, so $f(0) = 10$ is a local maximum. $f''(\pm\sqrt{3}) > 0$, so $f(\pm\sqrt{3}) = 1$ are local minima.

(b) $f(x) = x\sqrt{x-1}$, $f'(x) = \sqrt{x-1} + \frac{x}{2\sqrt{x-1}} = \frac{3x-2}{2\sqrt{x-1}} = 0$, so $x = \frac{2}{3}$, but this is not in the domain of f, so there are no maxima or minima.

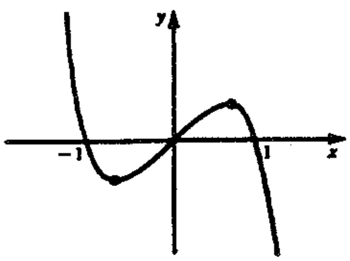
(c) $g(x) = \frac{x}{x^2+9}$, $g'(x) = \frac{x^2+9-2x^2}{(x^2+9)^2} = \frac{9-x^2}{(x^2+9)^2} = 0$, so $x = \pm 3$. g increases on $(-3,3)$ and g decreases on $(-\infty,-3)$ and $(3,\infty)$. Therefore, $g(-3) = -\frac{1}{6}$ is a local minimum and $g(3) = \frac{1}{6}$ is a local maximum.

(d) $g(x) = \frac{x}{(2x-3)^2}$, $g'(x) = \frac{(2x-3)^2 - 4x(2x-3)}{(2x-3)^4} = \frac{2x-3-4x}{(2x-3)^3} = \frac{-2x-3}{(2x-3)^3} = 0$, so $x = -\frac{3}{2}$, g decreases for $x < -\frac{3}{2}$ and g increases for $-\frac{3}{2} < x < \frac{3}{2}$. Therefore, $g(-\frac{3}{2}) = -\frac{1}{24}$ is a local minimum.

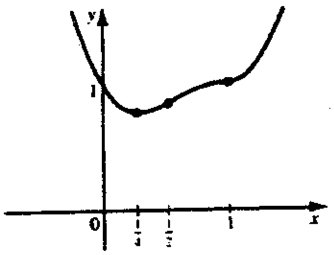
(e) $f(t) = \frac{t^2}{2t+5}$, $f'(t) = \frac{4t^2+10t-2t^2}{(2t+5)^2} = \frac{2t(t+5)}{(2t^2+5)} = 0$, so $x = 0$, $x = -5$. f increases on $(-\infty,-5)$ and $(0,\infty)$ and f decreases on $(-5,0)$. Therefore, $f(-5) = -5$ is a local maximum and $f(0) = 0$ is a local minimum.

(f) $f(t) = t + 3t^{\frac{2}{3}}$, $f'(t) = 1 + 2t^{-\frac{1}{3}} = 0$, so $t^{\frac{1}{3}} = -2$, thus $t = -8$. $f''(t) = -\frac{2}{3}t^{\frac{4}{3}}$, $f''(-8) < 0$, so $f(-8) = 4$ is a local maximum.

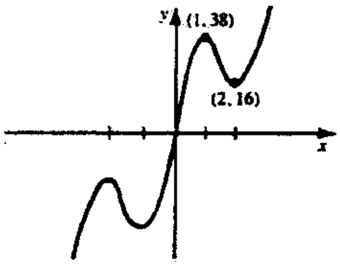
3. (a) $y = x - x^3$, $y' = 1 - 3x^2 = 0$, so $x = \pm\frac{1}{\sqrt{3}}$. y increases on $(-\infty, -\frac{1}{\sqrt{3}})$ and $(\frac{1}{\sqrt{3}},\infty)$ and y decreases on $(-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}})$. So $f(-\frac{1}{\sqrt{3}}) = -\frac{2\sqrt{3}}{9}$ is a local minimum and $f(\frac{1}{\sqrt{3}}) = \frac{2\sqrt{3}}{9}$ is a local maximum. $y'' = -6x$, so y is CU on $(-\infty,0)$ and y is CD on$(0,\infty)$. $(0,0)$ is an inflection point.



(b) $y = x^4 - 3x^3 + 3x^2 - x + 1$, $y' = 4x^3 - 9x^2 + 6x - 1 = (4x - 1)(x^2 - 2x + 1)$ $= (4x - 1)(x - 1)^2 = 0$, so $x = 1$, $x = \frac{1}{4}$. y decreases on $(-\infty, \frac{1}{4})$ and y increases on $(\frac{1}{4}, \infty)$. So $f(\frac{1}{4}) = \frac{229}{256}$ is a local minimum. $y'' = 12x^2 - 18x + 6 = 6(2x - 1)(x - 1)$, so y is CU on $(-\infty, \frac{1}{2})$ and $(1, \infty)$ and y is CD on $(\frac{1}{2}, 1)$. $(\frac{1}{2}, \frac{15}{16})$ and $(1,1)$ are inflection points.



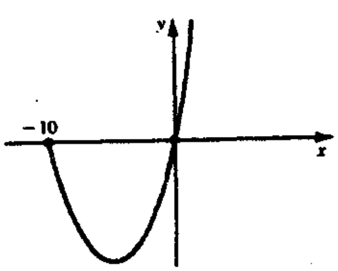
(c) $y = 3x^5 - 25x^3 + 60x$, $y' = 15x^4 - 75x^2 + 60 = 15(x^4 - 5x^2 + 4)$ $= 15(x^2 - 4)(x^2 - 1) = 0$, so $x = \pm 2$, $x = \pm 1$.
$y'' = 60x^3 - 150x = 30x(2x^2 - 5)$. When $x = -2$, $y'' < 0$, when $x = -1$, $y'' > 0$, when $x = 1$, $y'' < 0$, and when $x = 2$, $y'' > 0$. So $f(-2) = -16$ and $f(1) = 38$ are local maxima and $f(-1) = -38$ and $f(2) = 16$ are local minima. Since $y'' = 30x(2x^2 - 5)$, y is CU on $(-\sqrt{\frac{5}{2}}, 0)$ and $(\sqrt{\frac{5}{2}}, \infty)$ and y is CD on $(-\infty, -\sqrt{\frac{5}{2}})$ and $(0, \sqrt{\frac{5}{2}})$.



(d) $y = x\sqrt{10 + x}$, $y' = \frac{x}{2\sqrt{10 + x}} + \sqrt{10 + x} = \frac{3x + 20}{2\sqrt{10 + x}} = 0$, so $x = -\frac{20}{3}$.

Now y increases on $(-\frac{20}{3}, \infty)$ and y decreases on $(-10, -\frac{20}{3})$, so $f(-\frac{20}{3}) = -\frac{20\sqrt{30}}{9}$ is a local minimum. $y'' = \dfrac{6\sqrt{10 + x} - (3x + 20)\frac{1}{\sqrt{10 + x}}}{4(10 + x)} = \dfrac{6(10 + x) - (3x + 20)}{4(10 + x)^{\frac{3}{2}}}$

$= \dfrac{3x + 40}{4(10 + x)^{\frac{3}{2}}}$, so y is CU on $(-10, \infty)$.



Exercise 5.5

1. $y = 3x^5 - 10x^3 + 45x$.

A. The domain is R.

B. The y-intercept is 0 and the x-intercept is 0.

C. $f(-x) = -f(x)$ so the function is odd and symmetric about the origin.

D. There are no asymptotes, but $\lim_{x \to \infty} f(x) = \infty$ and $\lim_{x \to -\infty} f(x) = -\infty$.

E. $f'(x) = 15x^4 - 30x^2 + 45 = 15[(x^2-1)^2 + 2] > 0$ on R, so f increases on R.

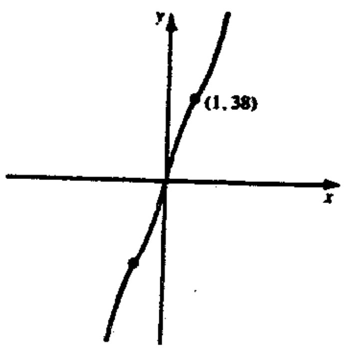
F. There are no local maximum or minimum values.

G. $f''(x) = 60x^3 - 60x = 60x(x^2 - 1)$

Interval	x	$x^2 - 1$	f''	f
$(-\infty, -1)$	$-$	$+$	$-$	CD
$(-1,0)$	$-$	$-$	$+$	CU
$(0,1)$	$+$	$-$	$-$	CD
$(1,\infty)$	$+$	$+$	$+$	CU

The inflection points are $(-1, -38)$, $(0,0)$, and $(1,38)$.

H.



2. $y = (x^2 - 1)^3$.

A. The domain is R.

B. The y-intercept is -1 and the x-intercepts are ± 1.

C. $f(-x) = f(x)$, so f is an even function and symmetric about the y-axis.

D. There are no asymptotes, but $\lim_{x \to \pm\infty} f(x) = \infty$.

E. $f'(x) = 6x(x^2 - 1)^2 > 0 \Leftrightarrow x > 0$, so f increases on $(0,\infty)$ and decreases on $(-\infty,0)$.

F. $f(0) = -1$ is a local minimum.

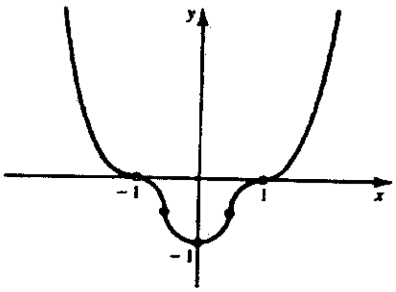
G. $f''(x) = 6(x^2 - 1)^2 + 24x^2(x^2 - 1) = 6(x^2 - 1)(5x^2 - 1)$.

Exercise 5.5

Interval	x^2-1	$5x^2-1$	f''	f
$(-\infty,-1)$	+	+	+	CU
$(-1,-\frac{1}{\sqrt{5}})$	−	+	−	CD
$(-\frac{1}{\sqrt{5}},\frac{1}{\sqrt{5}})$	−	−	+	CU
$(\frac{1}{\sqrt{5}},1)$	−	+	−	CD
$(1,\infty)$	+	+	+	CU

The inflection points are $(\pm 1,0)$ and $(\pm\frac{1}{\sqrt{5}}, -\frac{64}{125})$.

H.



3. $y = \frac{x-4}{x+4}$.

A. The domain is $(-\infty,-4)\bigcup(-4,\infty)$.

B. The y-intercept is -1 and the x-intercept is 4.

C. $f(-x) = \frac{-x-4}{-x+4}$, so there is no symmetry.

D. $\lim_{x\to\pm\infty}\frac{x-4}{x+4} = \lim_{x\to\pm\infty}\frac{1-\frac{4}{x}}{1+\frac{4}{x}} = 1$, so there is a horizontal asymptote at $y = 1$.

$\lim_{x\to -4^-}\frac{x-4}{x+4} = \infty$ and $\lim_{x\to -4^+}\frac{x-4}{x+4} = -\infty$, so there is a vertical asymptote at $x = -4$.

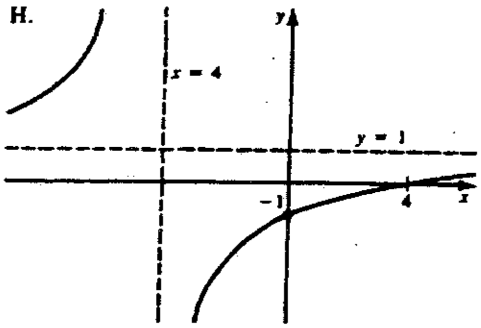
E. $f'(x) = \frac{8}{(x+4)^2} > 0$ on the domain, so f increases on $(-\infty,-4)$ and $(-4,\infty)$.

F. There are no local maximum or minimum values.

G. $f''(x) = -\frac{16}{(x+4)^3} > 0$ for $x < -4$, so f is CU on $(-\infty,-4)$ and f is CD on $(-4,\infty)$. There are no inflection points since $f(-4)$ does not exist.

Exercise 5.5

H.



4. $y = \dfrac{x^2}{x^2+3}$.

A. The domain is R.

B. The y-intercept is 0 and the x-intercept is 0.

C. $f(-x) = f(x)$, so it is an even function and it is symmetric about the y-axis.

D. $\lim\limits_{x\to\pm\infty} \dfrac{x^2}{x^2+3} = \lim\limits_{x\to\pm\infty} \dfrac{1}{1+\frac{3}{x^2}}$, so there is a horizonal asymptote at $y = 1$.

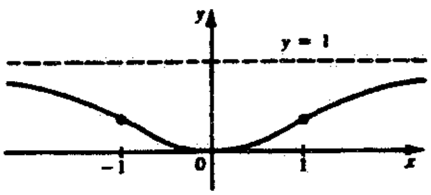
There are no vertical asymptotes.

E. $f'(x) = \dfrac{2x(x^2+3)-2x^3}{(x^2+3)^2} = \dfrac{6x}{(x^2+3)^2} < 0$ for $x < 0$, so f decreases on $(-\infty,0)$ and f increases on $(0,\infty)$.

F. $f(0) = 0$ is a local minimum.

G. $f''(x) = \dfrac{6(x^2+3)^2 - 24x^2(x^2+3)}{(x^2+3)^4} = \dfrac{18(1-x^2)}{(x^2+3)^3}$, so f is CU on $(-1,1)$ and f is CD on $(-\infty,-1)$ and $(1,\infty)$. The inflection points are $(\pm 1, \frac{1}{4})$.

H.



5. $y = \dfrac{x}{x^2-1}$.

A. The domain is $(-\infty,-1)\bigcup(-1,1)\bigcup(1,\infty)$.

B. The y-intercept is 0 and the x-intercept is 0.

C. $f(-x) = -\dfrac{x}{x^2-1} = -f(x)$, so f is an odd function and it is symmetric about the origin.

D. $\lim\limits_{x\to\pm\infty} \dfrac{x}{x^2-1} = 0$, so there is a horizontal asymptote at $y = 0$.

$\lim\limits_{x\to-1^-} f(x) = -\infty$, $\lim\limits_{x\to-1^+} f(x) = \infty$, $\lim\limits_{x\to 1^-} f(x) = -\infty$, and $\lim\limits_{x\to 1^+} f(x) = \infty$, so there are vertical asymptotes at $x = \pm 1$.

E. $f'(x) = \dfrac{x^2-1-2x^2}{(x^2-1)^2} = -\dfrac{x^2+1}{(x^2-1)^2} < 0$ on the domain, so f decreases on $(-\infty,-1)$, $(-1,1)$, and $(1,\infty)$.

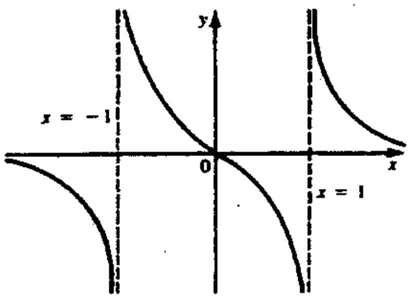
F. There are no maximum or minimum points.

G. $f''(x) = -\dfrac{2x(x^2-1)^2-(x^2+1)4x(x^2-1)}{(x^2-1)^4} = -\dfrac{2x\left[(x^2-1)-2x^2-2\right]}{(x^2-1)^3}$
$= \dfrac{2x(x^2+3)}{(x^2-1)^3}$.

Interval	x	x^2-1	f″	f
$(-\infty,-1)$	−	+	−	CD
$(-1,0)$	−	−	+	CU
$(0,1)$	+	−	−	CD
$(1,\infty)$	+	+	+	CU

The only inflection point is (0,0).

H.



6. $y = \dfrac{x}{(x-1)^2}$.

A. The domain is $(-\infty,1)\cup(1,\infty)$.

B. The y-intercept is 0 and the x-intercept is 0.

C. $f(-x) = -\dfrac{x}{(-x-1)^2}$, so there is no symmetry.

D. $\lim\limits_{x\to\pm\infty} \dfrac{x}{(x-1)^2} = 0$, so there is a horizontal asymptote at $y = 0$.

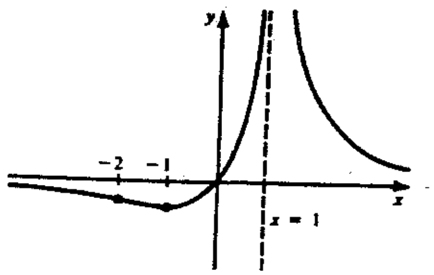
$\lim\limits_{x\to 1} \dfrac{x}{(x-1)^2} = \infty$, so there is a vertical asymptote at $x = 1$.

E. $f'(x) = \dfrac{(x-1)^2-2x(x-1)}{(x-1)^4} = -\dfrac{x+1}{(x-1)^3}$, so f increases on $(-1,1)$ and f decreases on $(-\infty,-1)$ and $(1,\infty)$.

F. $f(-1) = -\frac{1}{4}$ is a local minimum and $f(1)$ does not exist.

G. $f''(x) = \dfrac{-(x-1)^3 + 3(x+1)(x-1)^2}{(x-1)^6} = \dfrac{2(x+2)}{(x-1)^4} > 0$ for $x > -2$, so f is CU on $(-2,1)$ and $(1,\infty)$ and f is CD on $(-\infty,-2)$. The inflection point is $(-2,-\frac{2}{9})$.

H.



7. $y = \dfrac{1}{x^3 - x} = \dfrac{1}{x(x-1)(x+1)}$.

A. The domain is $(-\infty,-1)\cup(-1,0)\cup(0,1)\cup(1,\infty)$.

B. There are no intercepts.

C. $f(-x) = -f(x)$, so the function is odd and it is symmetric about the origin.

D. $\lim\limits_{x\to\pm\infty} \dfrac{1}{x^3 - x} = 0$, so there is a horizontal asymptote at $y = 0$.

$\lim\limits_{x\to -1^-} f(x) = -\infty$, $\lim\limits_{x\to -1^+} f(x) = \infty$, $\lim\limits_{x\to 0^-} f(x) = \infty$, $\lim\limits_{x\to 0^+} f(x) = -\infty$,

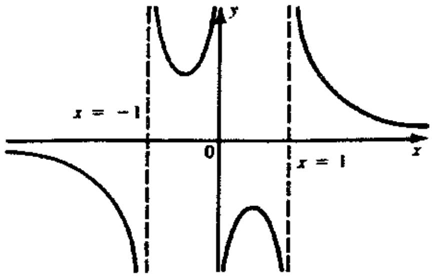
$\lim\limits_{x\to 1^-} f(x) = -\infty$, and $\lim\limits_{x\to 1^+} f(x) = -\infty$, so there are vertical asymptotes at $x = -1$, $x = 0$, and $x = 1$.

E. $f'(x) = \dfrac{1-3x^2}{(x^3-x)^2}$, so f increases on $(-\sqrt{\frac{1}{3}},\sqrt{\frac{1}{3}})$ and decreases on $(-\infty,-\sqrt{\frac{1}{3}})$ and $(\sqrt{\frac{1}{3}},\infty)$.

F. $f(-\sqrt{\frac{1}{3}}) = \frac{3\sqrt{3}}{2}$ is a local minimum and $f(\sqrt{\frac{1}{3}}) = -\frac{3\sqrt{3}}{2}$ is a local maximum.

G. $f''(x) = \dfrac{-6x(x^3-x)^2 - (1-3x^2)(2)(x^3-x)(3x^2-1)}{(x^3-x)^4} = \dfrac{12x^4 - 6x^2 + 2}{x^3(x^2-1)^3}$, so f is CU on $(-1,0)$ and $(1,\infty)$ and f is CD on $(-\infty,-1)$ and $(0,1)$. [Note that the numerator of f'' is always positive since $b^2-4ac<0$.] There are no inflection points.

H.



8. $y = \frac{x^2-1}{x^3}$.

A. The domain is $(-\infty,0)\bigcup(0,\infty)$.

B. There is no y-intercept and the x-intercepts are ± 1.

C. $f(-x) = -f(x)$, so the function is odd and it is symmetric about the origin.

D. $\lim_{x\to\pm\infty} f(x) = 0$, so there is a horizontal asymptote at $y = 0$.

$\lim_{x\to 0^-} f(x) = \infty$ and $\lim_{x\to 0^+} f(x) = -\infty$, so there is a vertical asymptote at $x = 0$.

E. $f'(x) = \frac{2x^4 - 3x^4 + 3x^2}{x^6} = \frac{3-x^2}{x^4} > 0$ for $x^2 < 3$, so f increases on $(-\sqrt{3},\sqrt{3})$ and f decreases on $(-\infty,-\sqrt{3})$ and $(\sqrt{3},\infty)$.

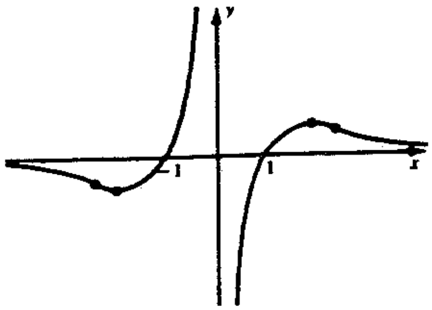
F. $f(-\sqrt{3}) = -\frac{2\sqrt{3}}{9}$ is a local minimum and $f(\sqrt{3}) = \frac{2\sqrt{3}}{9}$ is a local maximum.

G. $f''(x) = \frac{-2x^5 - 12x^3 + 4x^5}{x^8} = \frac{2(x^2-6)}{x^5}$.

Interval	x^5	x^2-6	f''	f
$(-\infty,-\sqrt{6})$	−	+	−	CD
$(-\sqrt{6},0)$	−	−	+	CU
$(0,\sqrt{6})$	+	−	−	CD
$(\sqrt{6},\infty)$	+	+	+	CU

The inflection points are $(-\sqrt{6}, -\frac{5}{6\sqrt{6}})$ and $(\sqrt{6}, \frac{5}{6\sqrt{6}})$.

H.



9. $y = x\sqrt{1-x^2}$.

A. The domain is $[-1,1]$.

B. The y-intercept is 0 and the x-intercepts are -1, 0, and 1.

C. $f(-x) = -f(x)$, so the function is odd and it is symmetric about the origin.

D. There are no asymptotes.

E. $f'(x) = -\frac{2x^2}{2\sqrt{1-x^2}} + \sqrt{1-x^2} = \frac{1-2x^2}{\sqrt{1-x^2}} > 0$ for $x^2 < \frac{1}{2}$, so f increases on

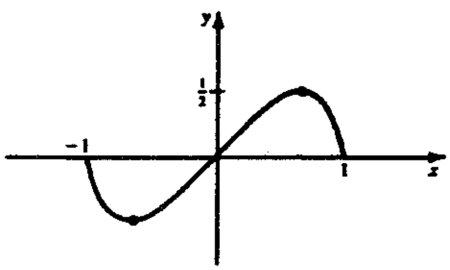
$(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$ and f decreases on $(-1,-\frac{1}{\sqrt{2}})$ and $(\frac{1}{\sqrt{2}},1)$.

F. $f(-\frac{1}{\sqrt{2}}) = -\frac{1}{2}$ is a local minimum and $f(\frac{1}{\sqrt{2}}) = \frac{1}{2}$ is a local maximum.

G. $f''(x) = \frac{-4x\sqrt{1-x^2} - (1-2x^2)\left(\frac{-x}{\sqrt{1-x^2}}\right)}{1-x^2} = \frac{-4x+4x^3+x-2x^3}{(1-x^2)^{\frac{3}{2}}} = \frac{x(2x^2-3)}{(1-x^2)^{\frac{3}{2}}}$,

so f is CU on $(-1,0)$ and f is CD on $(0,1)$. The inflection point is $(0,0)$.

H.



10. $y = \frac{x}{\sqrt{x^2-4}}$.

A. The domain is $(-\infty,-2)\cup(2,\infty)$.

B. There are no intercepts.

C. $f(-x) = -f(x)$, so the function is odd and it is symmetric about the origin.

D. $\lim_{x\to-\infty} f(x) = -1$ and $\lim_{x\to\infty} f(x) = 1$, so there are horizontal asymptotes at $y = \pm 1$.

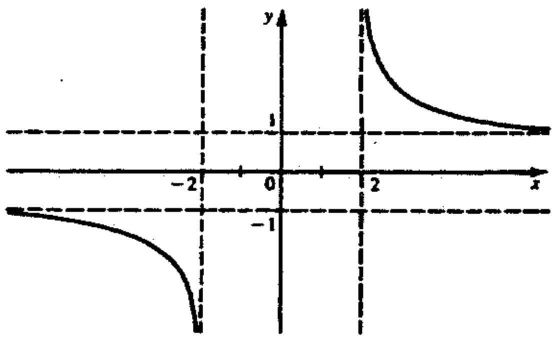
$\lim_{x\to-2^-} f(x) = -\infty$ and $\lim_{x\to 2^+} f(x) = \infty$, so there are vertical asymptotes at $x = \pm 2$.

E. $f'(x) = \dfrac{\sqrt{x^2-4} - \dfrac{x^2}{\sqrt{x^2-4}}}{x^2-4} = -\dfrac{4}{(x^2-4)^{\frac{3}{2}}} < 0$ on the domain, so f decreases on $(-\infty,-2)$ and $(2,\infty)$.

F. There are no maximum or minimum values.

G. $f''(x) = \dfrac{4(\frac{3}{2})(x^2-4)^{\frac{1}{2}}2x}{(x^2-4)^3} = \dfrac{12x}{(x^2-4)^{\frac{5}{2}}}$, so f is CD on $(-\infty,-2)$ and f is CU on $(2,\infty)$. There are no inflection points.

H.



11. $y = \dfrac{\sqrt{x}}{\sqrt{x}+1}$.

A. The domain is $[0,\infty)$.

B. The y-intercept is 0 and the x-intercept is 0.

C. There is no symmetry.

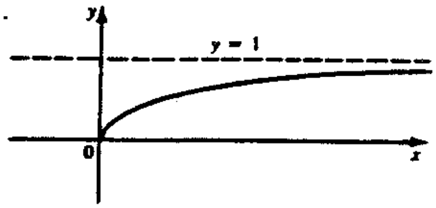
D. $\lim_{x\to\infty} f(x) = 1$, so the is a horizontal asymptote at $y = 1$ and there is no vertical asymptote.

E. $f'(x) = \dfrac{(\sqrt{x}+1)(\frac{1}{2\sqrt{x}}) - \sqrt{x}(\frac{1}{2\sqrt{x}})}{(\sqrt{x}+1)^2} = \dfrac{1}{2\sqrt{x}(\sqrt{x}+1)^2} > 0$, so f increases on $(0,\infty)$.

F. There are no maximum or minimum values.

G. $f''(x) = \dfrac{-\frac{1}{\sqrt{x}}(\sqrt{x}+1)^2 - 2(\sqrt{x}+1)}{4x(\sqrt{x}+1)^4} = \dfrac{-3-\frac{1}{\sqrt{x}}}{4x(\sqrt{x}+1)^3} = -\dfrac{(3\sqrt{x}+1)}{4x^{\frac{3}{2}}(\sqrt{x}+1)^3} < 0$ on the domain, so f is CD on $(0,\infty)$. There are no inflection points.

H.



12. $y = x - \sqrt[3]{x}$.

A. The domain is R.

B. The y-intercept is 0 and the x-intercepts are -1, 0, and 1.

C. $f(-x) = -f(x)$, so the function is odd and it is symmetric about the origin.

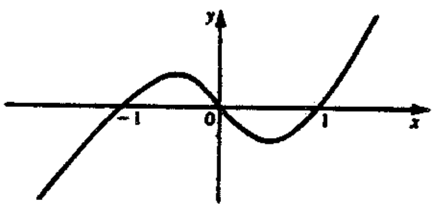
D. There are no asymptotes, but $\lim_{x\to-\infty} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = \infty$.

E. $f'(x) = 1 - \frac{1}{3}x^{-\frac{2}{3}} > 0$ when $x < \frac{1}{3^{3/2}}$, so f increases on $(-\infty, -\frac{1}{3\sqrt{3}})$ and $(\frac{1}{3\sqrt{3}},\infty)$ and f decreases on $(-\frac{1}{3\sqrt{3}},\frac{1}{3\sqrt{3}})$.

F. $f(-\frac{1}{3\sqrt{3}}) = \frac{2\sqrt{3}}{9}$ is a local maximum and $f(\frac{1}{3\sqrt{3}}) = -\frac{2\sqrt{3}}{9}$ is a local minimum.

G. $f''(x) = \frac{2}{9}x^{-\frac{5}{3}} > 0$ for $x > 0$, so f is CU on $(0,\infty)$ and f is CD on $(-\infty,0)$. There is an inflection point at $f(0) = 0$.

H.



Exercise 5.6

1. (a) $y = \frac{2x - x^2 - 1}{x} = 2 - x - \frac{1}{x}$, now $\lim_{x \to \pm\infty} [f(x) - (2 - x)] = \lim_{x \to \pm\infty} (-\frac{1}{x}) = 0$, so $y = 2 - x$ is the slant asymptote.

(b) $y = \frac{x^3 - 1}{x^2} = x - \frac{1}{x^2}$, now $\lim_{x \to \pm\infty} [f(x) - x] = \lim_{x \to \pm\infty} (-\frac{1}{x^2}) = 0$, so $y = x$ is the slant asymptote.

(c) $y = \frac{3x^3 + 4x + 2}{x + 1} = 3x + 1 + \frac{1}{x + 1}$, now $\lim_{x \to \pm\infty} [f(x) - (3x + 1)]$

$= \lim_{x \to \pm\infty} \frac{1}{x + 1} = 0$, so $y = 3x + 1$ is the slant asymptote.

(d) $y = \frac{4x^2}{2x + 1} = 2x - 1 + \frac{1}{2x + 1}$, now $\lim_{x \to \pm\infty} [f(x) - (2x - 1)] = \lim_{x \to \pm\infty} \frac{1}{2x + 1} = 0$, so $y = 2x - 1$ is the slant asymptote.

(e) $y = \frac{x^3 + 4x^2 + 5x + 16}{x^2 + 4} = x + 4 + \frac{x}{x^2 + 4}$, now $\lim_{x \to \pm\infty} [f(x) - (x + 4)]$

$= \lim_{x \to \pm\infty} \frac{x}{x^2 + 4} = 0$, so $y = x + 4$ is the slant asymptote.

(f) $y = \frac{x + x^2 - x^4}{x^3 - 1} = -x + \frac{x^2}{x^3 - 1}$, now $\lim_{x \to \pm\infty} [f(x) - (-x)] = \lim_{x \to \pm\infty} \frac{x^2}{x^3 - 1} = 0$, so $y = -x$ is the slant asymptote.

2. (a) $y = \frac{x^2 + 9}{x} = x + \frac{9}{x}$, now $\lim_{x \to \pm\infty} [f(x) - x] = \lim_{x \to \pm\infty} \frac{9}{x} = 0$, so $y = x$ is the slant asymptote.

A. The domain is $(-\infty, 0) \cup (0, \infty)$.

B. There are no intercepts.

C. $f(-x) = -f(x)$, so the function is odd and symmetric about the origin.

D. There are no horizontal asymptotes.

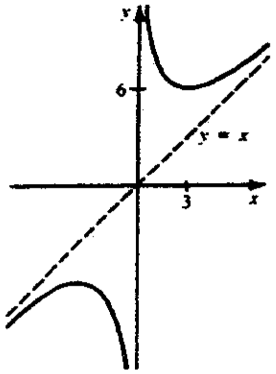
$\lim_{x \to 0^-} f(x) = -\infty$ and $\lim_{x \to 0^+} f(x) = \infty$, so $x = 0$ is a vertical asymptote.

E. $f'(x) = \frac{x^2 - 9}{x^2} > 0$ when $x^2 > 9 \Leftrightarrow |x| > 3$, so f increases on $(-\infty, -3)$ and $(3, \infty)$ and f decreases on $(-3, 0)$ and $(0, 3)$.

F. $f(-3) = -6$ is a local maximum and $f(3) = 6$ is a local minimum.

G. $f''(x) = \frac{18}{x^3}$, so f is CU on $(0,\infty)$ and f is CD on $(-\infty,0)$. There are no inflection points.

H.



(b) $y = \frac{x^2-2x-1}{x} = x-2-\frac{1}{x}$, now $\lim_{x\to\pm\infty}[f(x)-(x-2)] = \lim_{x\to\pm\infty}(-\frac{1}{x}) = 0$, so $y = x-2$ is a slant asymptote.

A. The domain is $(-\infty,0)\bigcup(0,\infty)$.

B. There is no y-intercept and the x-intercepts are $1\pm\sqrt{2}$.

C. There is no symmetry.

D. There are no horizontal asymptotes, but $\lim_{x\to\infty} f(x) = \infty$ and $\lim_{x\to-\infty} f(x) = -\infty$.

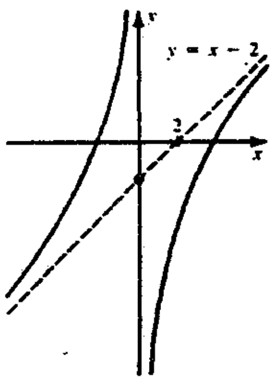
$\lim_{x\to 0^-} f(x) = \infty$ and $\lim_{x\to 0^+} f(x) = -\infty$, so there is a vertical asymptote at $x = 0$.

E. $f'(x) = \frac{x^2+1}{x^2} > 0$ on the domain, so f increases on $(-\infty,0)$ and $(0,\infty)$.

F. There are no maximum or minimum values.

G. $f''(x) = -\frac{2}{x^3} > 0$ when $x<0$, so f is CU on $(-\infty,0)$ and f is CD on $(0,\infty)$. There are no inflection points.

H.



(c) $y = \frac{x^3}{x^2-1} = x+\frac{x}{x^2-1}$, now $\lim_{x\to\pm\infty}[f(x)-x] = \lim_{x\to\pm\infty}\frac{x}{x^2-1} = 0$, so $y = x$ is the slant asymptote.

Exercise 5.6

A. The domain is $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$.

B. The y-intercept is 0 and the x-intercept is 0.

C. $f(-x) = -f(x)$, so the function is odd and it is symmetric about the origin.

D. There are no horizontal asymptotes.

$\lim_{x\to-1^-} f(x) = -\infty$, $\lim_{x\to-1^+} f(x) = \infty$, $\lim_{x\to1^-} f(x) = -\infty$, and $\lim_{x\to1^+} f(x) = \infty$, so there are vertical asymptotes at $x = \pm 1$.

E. $f'(x) = \dfrac{3x^4-3x^2-2x^4}{(x^2-1)^2} = \dfrac{x^2(x^2-3)}{(x^2-1)^2} > 0$ when $x^2 > 3$, so f increases on $(-\infty,-\sqrt{3})$ and $(\sqrt{3},\infty)$ and f decreases on $(-\sqrt{3},-1)$, $(-1,1)$, and $(1,\sqrt{3})$.

F. $f(-\sqrt{3}) = -\frac{3\sqrt{3}}{2}$ is a local maximum and $f(\sqrt{3}) = \frac{3\sqrt{3}}{2}$ is a local minimum.

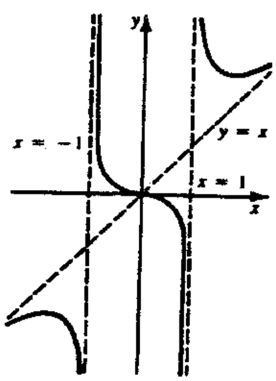
G. $$f''(x) = \frac{(4x^3-6x)(x^2-1)^2-(x^4-3x^2)(2)(x^2-1)(2x)}{(x^2-1)^4}$$

$$= \frac{4x^5-10x^3+6x-4x^5+12x^3}{(x^2-1)^3} = \frac{2x(x^2+3)}{(x^2-1)^3}.$$

Interval	x	$(x^2-1)^3$	f''	f
$(-\infty,-1)$	−	+	−	CD
$(-1,0)$	−	−	+	CU
$(0,1)$	+	−	−	CD
$(1,\infty)$	+	+	+	CU

The inflection point is (0,0).

H.



(d) $y = \frac{(x-1)^3}{x^2} = \frac{x^3 - 3x^2 + 3x - 1}{x^2} = x - 3 + \frac{3x-1}{x^2}$, now $\lim_{x\to\pm\infty}[f(x) - (x-3)]$

$= \lim_{x\to\pm\infty} \frac{3x-1}{x^2} = 0$, so $y = x - 3$ is the slant asymptote.

A. The domain is $(-\infty,0)\cup(0,\infty)$.

B. There is no y-intercept and the x-intercept is 1.

C. There is no symmetry.

D. There are no horizontal asymptotes.

$\lim_{x\to0^-} f(x) = -\infty$, and $\lim_{x\to0^+} f(x) = -\infty$, so there is a vertical asymptote at

$x = 0$.

E. $f'(x) = \frac{3x^2(x-1)^2 - 2x(x-1)^3}{x^4} = \frac{(x-1)^2(x+2)}{x^3}$.

Interval	x^3	$x+2$	f'	f
$(-\infty,-2)$	$-$	$-$	$+$	increases
$(-2,0)$	$-$	$+$	$-$	decreases
$(0,\infty)$	$+$	$+$	$+$	increases

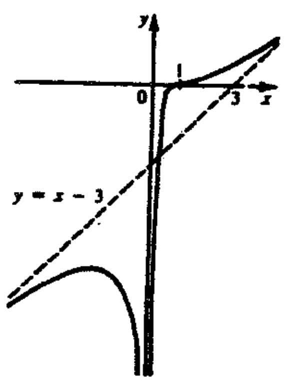
F. $f(-2) = -\frac{27}{4}$ is a local maximum.

G. $f''(x) = \frac{x^3[2(x-1)(x+2) + (x-1)^2] - 3x^2(x-1)^2(x+2)}{x^6}$

$= \frac{(x-1)[(2x^2+4x) + (x^2-x) - 3(x-1)(x+2)]}{x^4} = \frac{6(x-1)}{x^4}$, so f is CU on $(1,\infty)$ and

f is CD on $(-\infty,0)$ and $(0,1)$. The inflection point is $(1,0)$.

H.



3. $f(x) = \frac{x^3+1}{x} = x^2 + \frac{1}{x}$, now $\lim_{x\to\pm\infty}[f(x) - x^2] = \lim_{x\to\pm\infty}[(x^2+\frac{1}{x}) - x^2]$

$= \lim_{x\to\pm\infty} \frac{1}{x} = 0$, so f is asymptotic to $y = x^2$.

A. The domain is $(-\infty,0)\cup(0,\infty)$.

B. There is no y-intercept and the x-intercept is -1.

C. There is no symmetry.

D. $\lim_{x\to 0^-} f(x) = -\infty$ and $\lim_{x\to 0^+} f(x) = \infty$, so $x = 0$ is a vertical asymptote.

E. $f'(x) = \frac{2x^3-1}{x^2} > 0$ when $x^3 > \frac{1}{2}$, so f increases on $(\frac{1}{\sqrt[3]{2}},\infty)$ and f decreases on

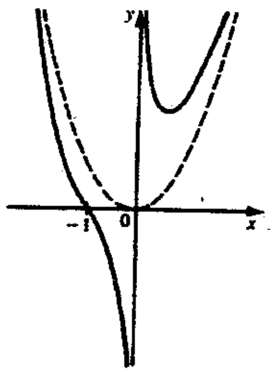
$(-\infty,0)$ and

$(0,\frac{1}{\sqrt[3]{2}})$.

F. $f(\frac{1}{\sqrt[3]{2}}) = \frac{3\sqrt[3]{2}}{2}$ is a local minimum.

G. $f''(x) = \frac{6x^4 - 4x^4 + 2x}{x^4} = \frac{2(x^3+1)}{x^3}$, so f is CU on $(-\infty,-1)$ and $(0,\infty)$ and f is CD on $(-1,0)$. The point of inflection is $(-1,0)$.

H.



5.7 Review Exercise

1. (a) $\lim_{x \to 4^+} \frac{2}{4-x} = -\infty.$ (b) $\lim_{x \to 4} \frac{6}{(x-4)^2} = \infty.$

(c) $\lim_{x \to -1^-} \frac{1}{(x+1)^3} = -\infty.$ (d) $\lim_{x \to 5^-} \frac{x+3}{x^2-4x-5} = -\infty.$

(e) $\lim_{x \to -2} \frac{x}{(x+2)^2} = -\infty.$ (f) $\lim_{x \to \infty} \frac{6-x}{6+5x} = \lim_{x \to \infty} \frac{\frac{6}{x}-1}{\frac{6}{x}+5} = \frac{0-1}{0+5} = -\frac{1}{5}.$

(g) $\lim_{x \to -\infty} \frac{x}{x^3-1} = \lim_{x \to -\infty} \frac{\frac{1}{x^2}}{1-\frac{1}{x^3}} = 0$ (h) $\lim_{x \to \infty} \frac{4x^2-3x+5}{2x^2+5x-4} = \lim_{x \to \infty} \frac{4-\frac{3}{x}+\frac{5}{x^2}}{2+\frac{5}{x}-\frac{4}{x^2}} = 2$

(i) $\lim_{x \to \infty} (x^4 - 2x^2) = \lim_{x \to \infty} x^2(x^2-2) = \infty.$

(j) Rationalize: $\lim_{x \to \infty} (\sqrt{x^2+x} - \sqrt{x^2-x}) = \lim_{x \to \infty} \frac{x^2+x-x^2+x}{\sqrt{x^2+x}+\sqrt{x^2-x}}$

$= \lim_{x \to \infty} \frac{2}{\sqrt{1+\frac{1}{x}}+\sqrt{1-\frac{1}{x}}} = \frac{2}{\sqrt{1+0}+\sqrt{1-0}} = 1.$

2. (a) $\lim_{x \to \frac{1}{2}^-} \frac{6x-1}{1-2x} = \infty,$ $\lim_{x \to \frac{1}{2}^+} \frac{6x-1}{1-2x} = -\infty,$ so there is a vertical asymptote at $x = \frac{1}{2}.$ $\lim_{x \to \pm\infty} \frac{6x-1}{1-2x} = -3,$ so there is a horizontal asymptote at $y = -3.$

(b) $y = \frac{1}{x^2+6x+9} = \frac{1}{(x+3)^2}.$ $\lim_{x \to -3} \frac{1}{(x+3)^2} = \infty,$ so there is a vertical asymptote at $x = -3.$

$\lim_{x \to \pm\infty} \frac{1}{(x+3)^2} = 0,$ so there is a horizontal asymptote at $y = 0.$

(c) $y = \frac{x}{2x^2-5x-3} = \frac{x}{(2x+1)(x-3)}.$ $\lim_{x \to -\frac{1}{2}^-} \frac{x}{(2x+1)(x-3)} = -\infty,$

$\lim_{x \to -\frac{1}{2}^+} \frac{x}{(2x+1)(x-3)} = \infty,$ $\lim_{x \to 3^-} \frac{x}{(2x+1)(x-3)} = -\infty,$ and $\lim_{x \to 3^+} \frac{x}{(2x+1)(x-3)} = \infty,$

so there are vertical asymptotes at $x = -\frac{1}{2}$ and $x = 3.$

$\lim_{x \to \pm\infty} \frac{x}{(2x+1)(x-3)} = 0,$ so there is a horizontal asymptote at $y = 0.$

(d) $\lim_{x \to 1^-} \frac{x^3}{x^3-1} = -\infty$ and $\lim_{x \to 1^+} \frac{x^3}{x^3-1} = \infty,$ so there is a vertical asymptote at $x = 1.$ $\lim_{x \to \pm\infty} \frac{x^3}{x^3-1} = 1,$ so there is a horizontal asymptote at $y = 1.$

3. (a) $y = 5x^3 + 12x^2 - 3x + 2$, $y' = 15x^2 + 24x - 3$, $y'' = 30x + 24 = 6(5x+4)$, so y is CU on $(-\frac{4}{5},\infty)$ and y is CD on $(-\infty, -\frac{4}{5})$. There is an inflection point at $(-\frac{4}{5}, \frac{238}{25})$.

(b) $y = x^4 - x^3 - 3x^2 + x - 12$, $y' = 4x^3 - 3x^2 - 6x + 1$,
$y'' = 12x^2 - 6x - 6 = 6(2x+1)(x-1)$, so y is CU on $(-\infty, -\frac{1}{2})$ and $(1,\infty)$ and y is CD on $(-\frac{1}{2},1)$. There are points of inflection at $(-\frac{1}{2}, -\frac{209}{16})$ and $(1, -14)$.

(c) $y = \dfrac{x^2}{x^2+4}$, $y' = \dfrac{2x(x^2+4) - 2x^3}{(x^2+4)^2} = \dfrac{8x}{(x^2+4)^2}$,

$y'' = \dfrac{8(x^2+4)^2 - 32x^2(x^2+4)}{(x^2+4)^4} = \dfrac{8x^2 + 32 - 32x^2}{(x^2+4)^3} = \dfrac{-8(3x^2-4)}{(x^2+4)^3}$, so y is CU on $(-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}})$ and y is CD on $(-\infty, -\frac{2}{\sqrt{3}})$ and $(\frac{2}{\sqrt{3}},\infty)$. There are inflection points at $(\pm\frac{2}{\sqrt{3}}, \frac{1}{4})$.

(d) $y = x + \frac{1}{\sqrt{x}}$, $x > 0$. $y' = 1 - \frac{1}{2}x^{-\frac{3}{2}}$, $y'' = \frac{3}{4}x^{-\frac{5}{2}}$, so y is CU on $(0,\infty)$. There are no inflection points.

4. (a) $f(x) = x^2 - x^3$, $f'(x) = 2x - 3x^2 = x(2-3x) = 0$, when $x = 0$, $x = \frac{2}{3}$. $f''(x) = 2 - 6x$, $f''(0) > 0$, so $f(0) = 0$ is a local minimum, $f''(\frac{2}{3}) < 0$, so $f(\frac{2}{3}) = \frac{4}{27}$ is a local maximum.

(b) $f(x) = 2x^3 + 15x^2 - 36x$, $f'(x) = 6x^2 + 30x - 36 = 6(x+6)(x-1) = 0$, when $x = -6$, $x = 1$. $f''(x) = 12x + 30$, $f''(-6) < 0$, so $f(-6) = 324$ is a local maximum, $f''(1) > 0$, so $f(1) = -19$ is a local minimum.

(c) $g(x) = \dfrac{x^2}{x-1}$, $g'(x) = \dfrac{2x^2 - 2x - x^2}{(x-1)^2} = \dfrac{x(x-2)}{(x-1)^2} = 0$, when $x = 0$, $x = 2$.

$g''(x) = \dfrac{(2x-2)(x-1)^2 - (x^2-2x)(2x-2)}{(x-1)^4} = \dfrac{2}{(x-1)^3}$, $g''(0) < 0$, so $g(0) = 0$ is a local maximum, $g''(2) > 0$, so $g(2) = 4$ is a local minimum.

(d) $g(x) = x + \sqrt{1-x}$, $g'(x) = 1 - \dfrac{1}{2\sqrt{1-x}} = 0$, when $\sqrt{1-x} = \frac{1}{2}$, so $x = \frac{3}{4}$.

$g''(x) = -\dfrac{1}{4(1-x)^{\frac{3}{2}}}$, $g''(\frac{3}{4}) < 0$, so $g(\frac{3}{4}) = \frac{5}{4}$ is a local maximum.

5. (a) $y = x^3 - 6x^2 + 9x$.

A. The domain is R.

B. The y-intercept is 0 and the x-intercepts are 0 and 3.

C. There is no symmetry

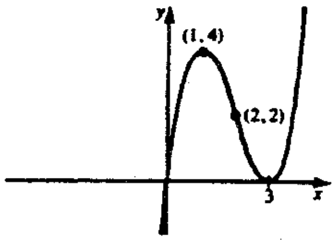
D. There are no asymptotes, but $\lim_{x\to\infty}(x^3-6x^2+9x) = \infty$ and $\lim_{x\to-\infty}(x^3-6x^2+9x) = -\infty$.

E. $y' = 3x^2-12x+9 = 3(x-1)(x-3)$, so y increases on $(-\infty,1)$ and $(3,\infty)$ and y decreases on $(1,3)$.

F. $f(1) = 4$ is a local maximum and $f(3) = 0$ is a local minimum.

G. $y'' = 6x-12 = 6(x-2)$ so y is CU on $(2,\infty)$ and y is CD on $(-\infty,2)$. There is a inflection point at $(2,2)$.

H.



(b) $y = x^3-x^4$

A. The domain is R.

B. The y-intercept is 0 and the x-intercepts are 0 and 1.

C. There is no symmetry.

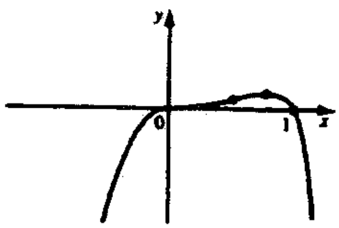
D. There are no asymptotes, but $\lim_{x\to\pm\infty} x^3(1-x) = -\infty$.

E. $y' = 3x^2-4x^3 = x^2(3-4x)$, so y increases on $(-\infty,\frac{3}{4})$ and y decreases on $(\frac{3}{4},\infty)$.

F. $f(\frac{3}{4}) = \frac{27}{256}$ is a local maximum.

G. $y'' = 6x-12x^2 = 6x(1-2x)$, so y is CU on $(0,\frac{1}{2})$ and CD on $(-\infty,0)$ and $(\frac{1}{2},\infty)$. There are inflection points at $(0,0)$ and $(\frac{1}{2},\frac{1}{16})$.

H.



(c) $y = \frac{2}{4+x}$.

A. The domain is $(-\infty,-4)\bigcup(-4,\infty)$.

B. The y-intercept is $\frac{1}{2}$ and there is no x-intercept.

C. There is no symmetry.

D. $\lim_{x\to\pm\infty}\frac{2}{4+x} = 0$, so there is a horizontal asymptote at $y = 0$.

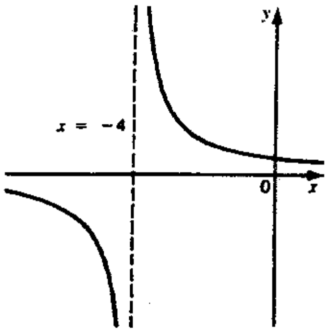
$\lim_{x \to -4^-} \frac{2}{4+x} = -\infty$ and $\lim_{x \to -4^+} \frac{2}{4+x} = \infty$, so there is a vertical asymptote at $x = -4$.

E. $y' = -\frac{2}{(4+x)^2} < 0$, on the domain, so y decreases on $(-\infty, -4)$ and $(-4, \infty)$.

F. There are no maximum or minimum values.

G. $y'' = \frac{4}{(4+x)^3}$, so y is CU on $(-4, \infty)$ and y is CD on $(-\infty, -4)$. There are no inflection points.

H.



(d) $y = \frac{1-x^2}{1+x^2}$.

A. The domain is R.

B. The y-intercept is 1 and the x-intercepts are ± 1.

C. $f(-x) = f(x)$, so the function is even and it is symmetric about the y-axis.

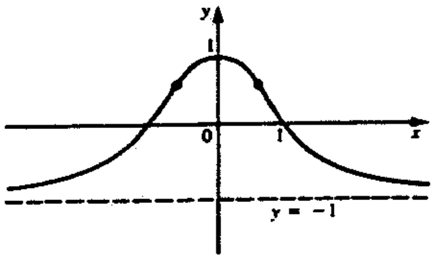
D. $\lim_{x \to \pm\infty} \frac{1-x^2}{1+x^2} = -1$, so there is a horizontal asymptote at $y = -1$. There are no vertical asymptotes.

E. $y' = \frac{-2x(1+x^2) - 2x(1-x^2)}{(1+x^2)^2} = -\frac{4x}{(1+x^2)^2}$, so y increases on $(-\infty, 0)$ and y decreases on $(0, \infty)$.

F. (0,1) is a local maximum.

G. $y'' = -\frac{4(1+x^2)^2 - 16x^2(1+x^2)}{(1+x^2)^4} = \frac{4(3x^2-1)}{(1+x^2)^3}$, so y is CU on $(-\infty, -\frac{1}{\sqrt{3}})$ and $(\frac{1}{\sqrt{3}}, \infty)$ and y is CD on $(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$. The inflection points are $(\pm\frac{1}{\sqrt{3}}, \frac{1}{2})$.

H.



(e) $y = \frac{1+x^2}{1-x^2}$.

A. The domain is $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$.

B. The y-intercept is 1 and there are no x-intercepts.

C. $f(-x) = f(x)$, so the function is even and symmetric about the y-axis.

D. $\lim_{x\to\pm\infty}\frac{1+x^2}{1-x^2} = -1$, so there is a horizontal asymptote at $y = -1$.

$\lim_{x\to-1^-}\frac{1+x^2}{1-x^2} = -\infty$, $\lim_{x\to-1^+}\frac{1+x^2}{1-x^2} = \infty$, $\lim_{x\to1^-}\frac{1+x^2}{1-x^2} = \infty$, and

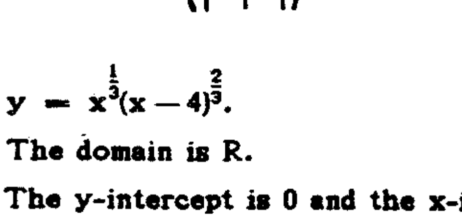
$\lim_{x\to1^+}\frac{1+x^2}{1-x^2} = -\infty$, so there are vertical asymptotes at $x = -1$ and $x = 1$.

E. $y' = \frac{2x(1-x^2)+2x(1+x^2)}{(1-x^2)^2} = \frac{4x}{(1-x^2)^2}$, so y increases on (0,1) and $(1,\infty)$ and y decreases on $(-\infty,-1)$ and $(-1,0)$.

F. $f(0) = 1$ is a local minimum.

G. $y'' = \frac{4(1-x^2)^2+16x^2(1-x^2)}{(1-x^2)^4} = \frac{4(3x^2+1)}{(1-x^2)^3}$, so y is CU on $(-1,1)$ and y is CD on $(-\infty,-1)$ and $(1,\infty)$. There are no inflection points.

H.



(f) $y = x^{\frac{1}{3}}(x-4)^{\frac{2}{3}}$.

A. The domain is R.

B. The y-intercept is 0 and the x-intercepts are 0 and 4.

C. There is no symmetry.

D. There are no intercepts but $\lim_{x\to\infty} x^{\frac{1}{3}}(x-4)^{\frac{2}{3}} = \infty$ and $\lim_{x\to-\infty} x^{\frac{1}{3}}(x-4)^{\frac{2}{3}} = -\infty$.

E. $y' = \frac{1}{3}x^{-\frac{2}{3}}(x-4)^{\frac{2}{3}} + \frac{2}{3}x^{\frac{1}{3}}(x-4)^{-\frac{1}{3}} = \frac{3x-4}{3x^{\frac{2}{3}}(x-4)^{\frac{1}{3}}} = \frac{3x-4}{3\left[x^2(x-4)\right]^{\frac{1}{3}}}$.

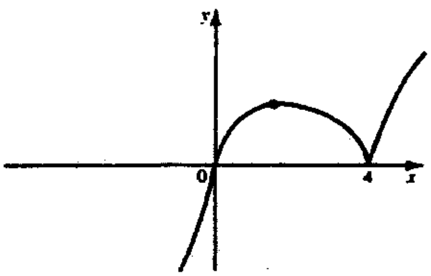
Interval	$3x-4$	$x-4$	y'	y
$(-\infty,\frac{4}{3})$	$-$	$-$	$+$	increasing
$(\frac{4}{3},4)$	$+$	$-$	$-$	decreasing
$(4,\infty)$	$+$	$+$	$+$	increasing

F. $g(\frac{4}{3}) = \frac{4}{3}\sqrt[3]{4}$ is a local maximum and $f(4) = 0$ is a local minimum.

G. $$y'' = \frac{1}{3}\,\frac{3(x^3-4x^2)^{\frac{1}{3}} - (3x-4)\frac{1}{3}(3x^2-8x)\left[x^2(x-4)\right]^{-\frac{2}{3}}}{\left[x^2(x-4)\right]^{\frac{2}{3}}}$$

$$= \frac{9x^3-36x^2-(9x^3-36x^2-32x)}{9\left[x^2(x-4)\right]^{\frac{4}{3}}} = \frac{-32}{9x^{\frac{5}{3}}(x-4)^{\frac{4}{3}}}$$ so y is CU on $(-\infty,0)$ and y is CD on $(0,4)$ and $(4,\infty)$. There is a point of inflection at $(0,0)$.

H.



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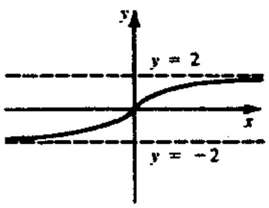

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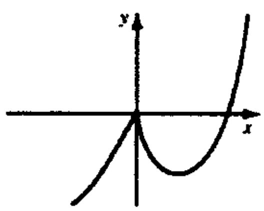


5.8 Chapter 5 Test

1. (a) $\lim_{x\to\infty} \frac{6x^3 - 3x + 1}{2x^3 + x^2 - 5} = \lim_{x\to\infty} \frac{6 - \frac{3}{x^2} + \frac{1}{x^3}}{2 + \frac{1}{x} - \frac{5}{x^3}} = \frac{6 - 0 + 0}{2 + 0 - 0} = 3.$

(b) $\lim_{x\to -2^+} \frac{x-1}{x^2-4} = \infty.$

2. $\lim_{x\to \pm\infty} \frac{1-2x}{3x+5} = -\frac{2}{3}$, so there is a horizontal asymptote at $y = -\frac{2}{3}$.

$\lim_{x\to -\frac{5}{3}^-} \frac{1-2x}{3x+5} = -\infty$ and $\lim_{x\to -\frac{5}{3}^+} \frac{1-2x}{3x+5} = \infty$, so there is a vertical asymptote at $x = -\frac{5}{3}$.

3. (a) $y = \frac{x}{(x+1)^2}$, $y' = \frac{x+1-2x}{(x+1)^3} = \frac{1-x}{(x+1)^3}$,

$y'' = \frac{-(x+1)^3 - 3(1-x)(x+1)^2}{(x+1)^6} = \frac{2x-4}{(x+1)^4}$, so y is CU on $(2,\infty)$ and y is CD on $(-\infty,2)$.

(b) There is a point of inflection at $(2,\frac{2}{9})$.

4. $y = 2 - 12x + 9x^2 - 2x^3$

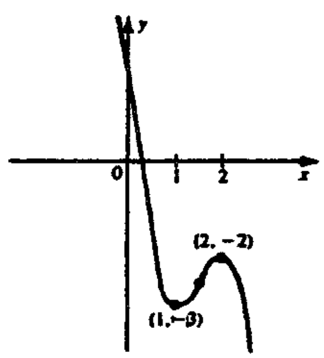
(a) $y' = -12 + 18x - 6x^2 = -6(x-1)(x-2)$, so y increases on (1,2) and y decreases on $(-\infty,1)$ and $(2,\infty)$.

(b) There is a local maximum at $f(2) = -2$ and a local minimum at $f(1) = -3$.

(c) $y'' = 18 - 12x$, so y is CU on $(-\infty,\frac{3}{2})$ and y is CD on $(\frac{3}{2},\infty)$.

(d) There is an inflection point at $(\frac{3}{2}, -\frac{5}{2})$.

(e)



5. $y = \frac{x}{x^2 - 9}$

(a) The domain is $(-\infty, -3) \cup (-3,3) \cup (3,\infty)$.

(b) The y-intercept is 0 and the x-intercept is 0.

(c) $f(x) = -f(x)$, so the function is odd and it is symmetric about the origin.

(d) $\lim_{x \to \pm\infty} \frac{x}{x^2 - 9} = 0$, so there is a horizontal asymptote at $y = 0$.

$\lim_{x \to -3^-} \frac{x}{x^2 - 9} = -\infty$, $\lim_{x \to -3^+} \frac{x}{x^2 - 9} = \infty$, $\lim_{x \to 3^-} \frac{x}{x^2 - 9} = -\infty$, and

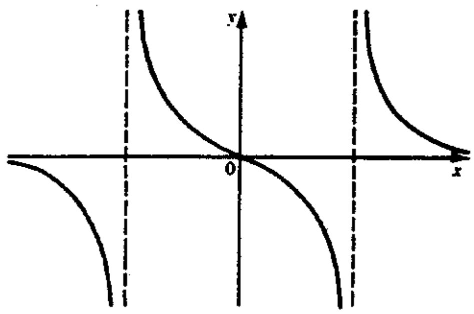
$\lim_{x \to 3^+} \frac{x}{x^2 - 9} = \infty$, so there are vertical asymptotes at $x = -3$ and $x = 3$.

(e) $y' = \frac{x^2 - 9 - 2x^2}{(x^2 - 9)^2} = -\frac{(x^2 + 9)}{(x^2 - 9)} < 0$ on the domain, so y decreases on $(-\infty, -3)$, $(-3,3)$, and $(3, -\infty)$.

(f) There are no maximum or minimum values.

(g) $y'' = \frac{-2x(x^2 - 9) + (x^2 + 9)(4x)}{(x^2 - 9)^3} = \frac{2x(x^2 + 27)}{(x^2 - 9)^3}$, so y is CD on $(-\infty, -3)$ and $(0,3)$ and y is CU on $(-3,0)$ and $(3,\infty)$. There is a point of inflection at $(0,0)$.

(h)



EXERCISE 1

1. (a) $\frac{\pi}{6} = \frac{180}{6} = 30°$ (b) $\frac{-3\pi}{2} = \frac{-3\times 180}{2} = -270°$

   (c) $\frac{5\pi}{4} = \frac{5\times 180}{4} = 225°$ (d) $3\pi = 3\times 180 = 540°$

   (e) $4 = \frac{4\times 180}{\pi} \doteq 229°$ (f) $-\frac{3}{4} = \frac{-0.75\times 180}{\pi} \doteq -43°$

   (g) $-12 = \frac{-12\times 180}{\pi} = -688°$

2. (a) $45° = \frac{45\times\pi}{180} = \frac{\pi}{4} \doteq 0.79$ (b) $315° = \frac{315\times\pi}{180} = \frac{7\pi}{4} \doteq 5.50$

   (c) $-210° = \frac{-210\times\pi}{180} = -\frac{7\pi}{6} \doteq 3.67$ (d) $570° = \frac{570\times\pi}{180} = \frac{19\pi}{6} \doteq 9.95$

   (e) $2° = \frac{2\times\pi}{180} = \frac{\pi}{90} \doteq 0.03$ (f) $-28° = \frac{-28\times\pi}{180} = \frac{-7\pi}{45} \doteq -0.49$

   (g) $601° = \frac{601\pi}{180} \doteq 10.5$

3. (a) $a = r\theta = 10\times 2.5 = 25$ (b) $\theta = \frac{a}{r} = \frac{12}{10} = 1.2$

4. $72° = \frac{72\times\pi}{180} = \frac{2\pi}{5} = \theta$. Therefore $r = \frac{a}{\theta} = \frac{32}{0.4\pi} = \frac{80}{\pi} \doteq 25.46$

EXERCISE 2

1. $r^2 = x^2 + y^2 = 9 + 16 = 25$. Therefore $r = 5$ and
   $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = \frac{4}{3}$, $\csc\theta = \frac{5}{4}$, $\sec\theta = \frac{5}{3}$, $\cot\theta = \frac{3}{4}$.

2. $r^2 = 4 + 1 = 5$. Therefore $r = \sqrt{5}$ and
   $\sin\theta = \frac{-1}{\sqrt{5}}$, $\cos\theta = \frac{-2}{\sqrt{5}}$, $\tan\theta = \frac{1}{2}$.

3. $r^2 = 25 + 144$. Therefore $r = 13$ and
   $\csc\theta = -\frac{13}{12}$, $\sec\theta = \frac{13}{5}$, $\cot\theta = -\frac{5}{12}$.