EXERCISE 1

1. (a) Area $=\frac{1}{2}(\frac{4\pi}{6}-\frac{\pi}{6})[f(\frac{\pi}{6})+f(\frac{4\pi}{6})]=\frac{\pi}{4}(\sin\frac{\pi}{6}+\sin\frac{4\pi}{6})=\frac{\pi}{4}(\frac{1}{2}+\frac{\sqrt{3}}{2})=\frac{\pi}{8}(1+\sqrt{3})$

   (b) Area $=\frac{1}{2}(\frac{\pi}{2}-\frac{\pi}{6})(\sin^{-1}\frac{\pi}{2}+\sin^{-1}\frac{\pi}{6})=\frac{1}{2}(\frac{2\pi}{6})(1+\frac{1}{2})=\frac{\pi}{4}$

   (c) Area $=\frac{1}{2}(2-1)(\frac{1}{2}+1)=\frac{3}{4}$

EXERCISE 2

1. (a) $\sum_{i=1}^{5}(i^2+1)=(1^2+1)+(2^2+1)+(3^2+1)+(4^2+1)+(5^2+1)=60$

   (b) $\sum_{i=1}^{4}\frac{i}{4}f(i)=\frac{1}{4}f(1)+\frac{1}{2}f(2)+\frac{3}{4}f(3)+f(4)$

   (c) $\sum_{i=1}^{n}\frac{3}{n}f(1+\frac{3}{4}i)=\frac{3}{n}f(1+\frac{3}{4})+\frac{3}{n}f(1+\frac{6}{4})+\frac{3}{n}f(1+\frac{9}{4})+\cdots+\frac{3}{n}f(1+\frac{3n}{4})$

2. (a) The general term is $1+(n-1)3=3n-2$. Therefore $\sum_{i=1}^{6}(3i-2)$ is the series.

   (b) The general term is $(-1)^{n-1}$. Therefore $\sum_{i=1}^{7}(-1)^{i-1}$ is the series.

   (c) The general term is x^n. Therefore $\sum_{i=1}^{n}x^i$ is the series.

   (d) The general term is $\frac{n}{6}f(\frac{n}{6})$. Therefore $\sum_{i=1}^{6}\frac{i}{6}f(\frac{i}{6})$ is the series.

   (e) The general term is $\frac{r}{n}f(\frac{2r-2}{n})$. Therefore $\sum_{i=1}^{n}\frac{i}{n}f(\frac{2i-2}{n})$ is the series.

3. (a) $\sum_{1=1}^{n}(2+i)^2=\sum_{i=1}^{n}(4+4i+i^2)=\sum_{i=1}^{n}i^2+4\sum_{i=1}^{n}i+4n$

   (b) $\sum_{i=1}^{20}(3i^2-12i)=3\sum_{i=1}^{20}i^2-12\sum_{i=1}^{20}i$

3. (c) $\sum_{i=1}^{n}(2i^3-3i^2+5i-12)=2\sum_{i=1}^{n}i^3-3\sum_{i=1}^{n}i^2+5\sum_{i=1}^{n}i-12n$

EXERCISE 3

1. (a) $3+7+11+\cdots+(4n-1)=\sum_{i=1}^{n}(4n-1)=4\frac{n(n+1)}{2}-n=n(2n+1)$

or $S_n=\frac{n}{2}[2(3)+(n-1)4]=\frac{n}{2}[6+4n-4]=\frac{n}{2}[4n+2]=n(2n+1)$

(b) $1+3+9+\cdots+3^{n-1}=\frac{1(3^n-1)}{3-1}=\frac{1}{2}(3^n-1)$

(c) $\sum_{i=1}^{n}(3i^2-i)=3\sum_{i=1}^{n}i^2-\sum_{i=1}^{n}i=3\frac{n(n+1)(2n+1)}{6}-\frac{n(n+1)}{2}$

$=\frac{1}{2}n(n+1)(2n+1-1)=n^2(n+1)$

(d) $\sum_{i=1}^{n}(2i^3+3i-2)=2\sum_{i=1}^{n}i^3+3\sum_{i=1}^{n}i-2n=2\frac{n^2(n+1)^2}{4}+3\frac{n(n+1)}{2}-2n$

$=\frac{n^2(n+1)^2+3n(n+1)-4n}{2}=\frac{n}{2}(n^3+2n^2+4n-1)$

(e) $\sum_{i=1}^{20}(i+3)=\sum_{i=1}^{20}i+3(20)=\frac{20(21)}{2}+60=210+60=270$

(f) $\sum_{i=41}^{100}(i^3-2i)=\sum_{i=1}^{100}(i^3-2i)-\sum_{i=1}^{40}(i^3-2i)$

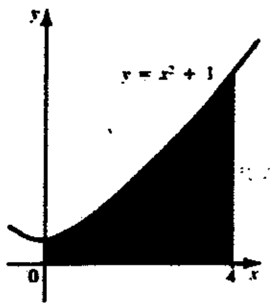
$=\frac{100^2\times101^2}{4}-2\frac{100\times101}{2}-\left(\frac{40^2\times41^2}{4}-2\frac{40\times41}{2}\right)=24\,821\,640$

EXERCISE 10.1

1. (a) $F(x)=\frac{x^3}{3}+x.$

$A(4)=F(4)-F(0)$

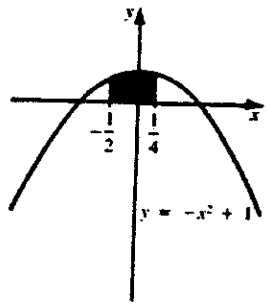
$=\frac{64}{3}+4-0=\frac{76}{3}$



(b) $F(x)=-\frac{x^3}{3}+x.$

$A(\frac{1}{4})=F(\frac{1}{4})-F(-\frac{1}{2})$

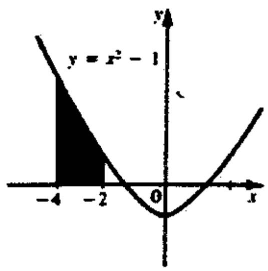
$=-\frac{1}{192}+\frac{1}{4}-\frac{1}{24}+\frac{1}{2}=\frac{135}{192}=\frac{45}{64}$



(c) $F(x)=\frac{x^3}{3}-x.$

$A(-2)=F(-2)-F(-4)$

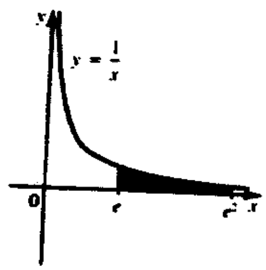
$=-\frac{8}{3}+2+\frac{64}{3}-4=\frac{50}{3}$



(d) $F(x)=\ln x.$

$A(e^2)=F(e^2)-F(e)$

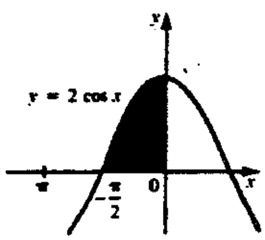
$=\ln e^2-\ln e=2-1=1$



(e) $F(x)=2\sin x.$

$A(0)=F(0)-F(-\frac{\pi}{2})$

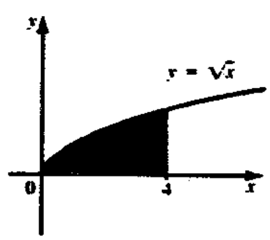
$=2(0)-2(-1)=2$



(f) $F(x)=\frac{2}{3}\sqrt{x^3}.$

$A(4)=F(4)-F(0)$

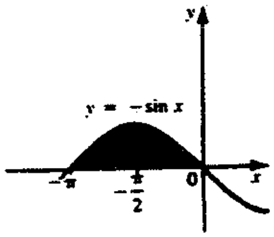
$=\frac{16}{3}-0=\frac{16}{3}$



(g) $F(x) = \cos x.$

$A(0) = F(0) - F(-\pi)$

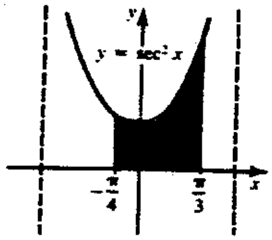
$= 1 - (-1) = 2$



(h) $F(x) = \tan x.$

$A(\frac{\pi}{3}) = F(\frac{\pi}{3}) - F(-\frac{\pi}{4})$

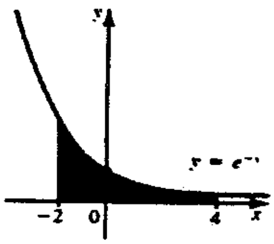
$= \sqrt{3} - (-1) = 1 + \sqrt{3}$



(i) $F(x) = -e^{-x}.$

$A(4) = F(4) - F(-2)$

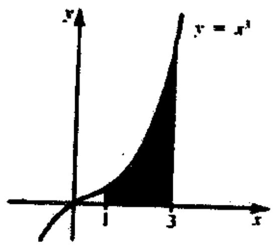
$= -e^{-4} + e^{2} = \frac{e^{6} - 1}{e^{4}}$



(j) $F(x) = \frac{1}{4}x^{4}.$

$A(3) = F(3) - F(1)$

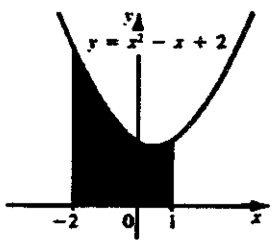
$= \frac{81}{4} - \frac{1}{4} = 20$



(k) $F(x) = \frac{1}{3}x^{3} - \frac{1}{2}x^{2} + 2x.$

$A(1) = F(1) - F(-2)$

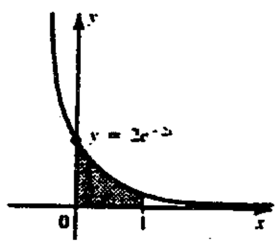
$= \frac{1}{3} - \frac{1}{2} + 2 + \frac{8}{3} + 2 + 4 = 10\frac{1}{2}$



(l) $F(x) = -e^{-2x}.$

$A(1) = F(1) - F(0)$

$= -e^{-2} + 1 = \frac{e^{2} - 1}{e^{2}}$



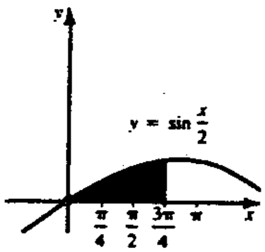
Exercise 10.1

(m) $F(x) = -2\cos\frac{x}{2}$.

$A(x) = F(\frac{3\pi}{4}) - F(0)$

$= -2\cos\frac{3\pi}{8} + 2\cos 0$

$= -2\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}} + 2$

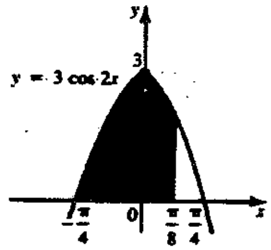


(n) $F(x) = \frac{3}{2}\sin 2x$.

$A(\frac{\pi}{8}) = F(\frac{\pi}{8}) - F(-\frac{\pi}{4})$

$= \frac{3}{2}(\sin\frac{\pi}{4} - \sin(-\frac{\pi}{2})$

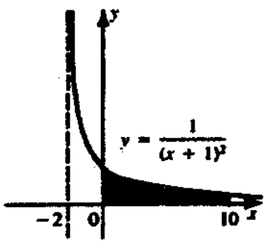
$= \frac{3(1+\sqrt{2})}{2\sqrt{2}}$



(o) $F(x) = \frac{-1}{x+1}$.

$A(10) = F(10) - F(0)$

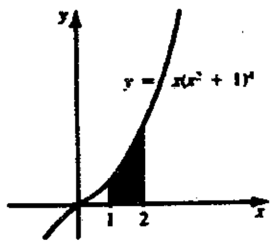
$= -\frac{1}{11} - (-1) = \frac{10}{11}$



(p) $F(x) = \frac{(x^2+1)^5}{10}$

$A(2) = F(2) - F(1)$

$= \frac{3125}{10} - \frac{32}{10} = \frac{3093}{10}$



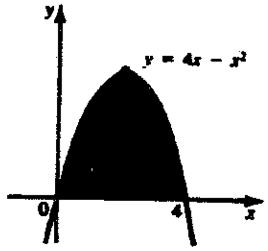
2. (a) $4x - x^2 = 0 \Rightarrow x(4-x) = 0$.

The x-intercepts are 0 and 4.

$F(x) = 2x^2 - \frac{x^3}{3}$.

$A(4) = F(4) - F(0)$

$= 32 - \frac{64}{3} = 10\frac{2}{3}$



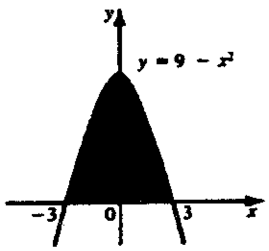
(b) $9-x^2=0 \Rightarrow (3-x)(3+x)=0.$

The x-intercepts are -3 and 3.

$F(x)=9x-\frac{1}{3}x^3.$

$A(3)=F(3)-F(-3)$

$=27-9+27-9=36$

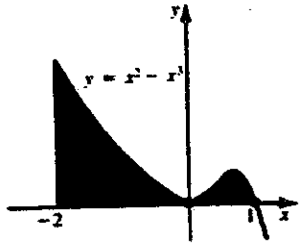


(c) $F(x)=\frac{1}{3}x^3-\frac{1}{4}x^4.$

Two regions are determined

$A(1)=F(1)-F(0)$

$=\frac{1}{3}-\frac{1}{4}=\frac{1}{12}$



$A(0)=F(0)-F(-2)$

$=-(-\frac{8}{3}-4)=\frac{20}{3}$

Total area is $\frac{1}{12}+\frac{20}{3}=6\frac{3}{4}$

(d) $x^2-x^4=0 \Rightarrow x^2(1-x^2)=0.$

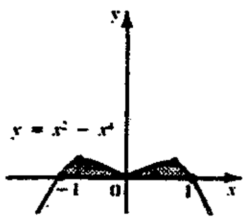
The x-intercepts are -1, 0 and 1.

Two symmetric regions are determined.

The area between 0 and 1 is above the x-axis.

as is the area between -1 and 0.

$F(x)=\frac{1}{3}x^3-\frac{1}{5}x^5.$



$A(1)=F(1)-F(0)$

$=\frac{1}{3}-\frac{1}{5}=\frac{2}{16}$

Total area is $\frac{4}{16}$

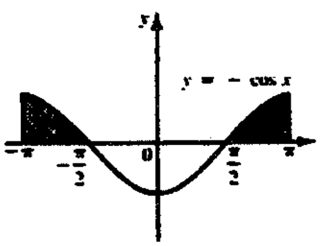
(e) Two symmetric regions are determined

$F(x)=-\sin x.$

$A(\pi)=F(\pi)-F(\frac{\pi}{2})$

$=0-(-1)=1$

The required area is $2(1)=2$



(f) $10 - 11x - 6x^2 = 0 \Rightarrow (5 + 2x)(2 - 3x) = 0$.

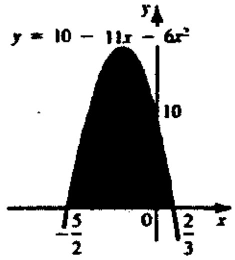
The x-intercepts are $-\frac{5}{2}$ and $\frac{2}{3}$.

$F(x) = 10x - \frac{11}{2}x^2 - 2x^3$.

$A(\frac{2}{3}) = F(\frac{2}{3}) - F(-\frac{5}{2})$

$= \frac{20}{3} - \frac{22}{9} - \frac{16}{27} + 25 + \frac{275}{8} - \frac{125}{4}$

$= \frac{6859}{216}$



(g) $x^3 - 3x^2 - 9x + 27 = 0$

$\Rightarrow x^2(x - 3) - 9(x - 3) = 0$

$\Rightarrow (x - 3)(x^2 - 9) = 0$.

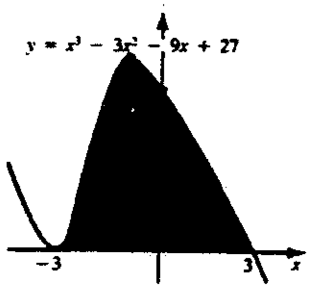
The x-intercepts are -3 and 3.

$F(x) = \frac{1}{4}x^4 - x^3 - \frac{9}{2}x^2 + 27x$.

$A(3) = F(3) - F(-3)$

$= \frac{81}{4} - 27 - \frac{81}{2} + 81 - \frac{81}{4} - 27 + \frac{81}{2} + 81$

$= 108$



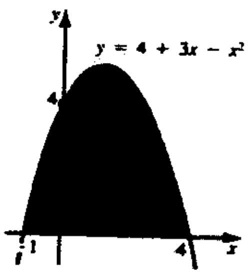
(h) $4 + 3x - x^2 = 0 \Rightarrow (4 - x)(1 + x) = 0$.

The x-intercepts are -1 and 4.

$F(x) = 4x + \frac{3}{2}x^2 - \frac{1}{3}x^3$.

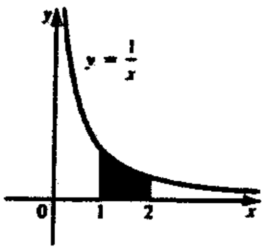
$A(4) = F(4) - F(-1)$

$= 16 + 24 - \frac{64}{3} + 4 - \frac{3}{2} - \frac{1}{3} = \frac{125}{6}$

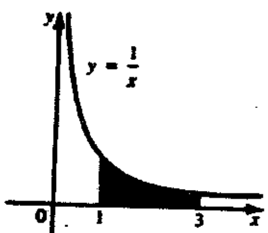


3. $F(x) = \ln x$. $A(b) = F(b) - F(1) = \ln b - \ln 1 = \ln b$.

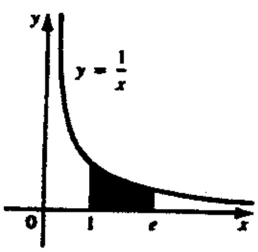
(a) $A(2) = \ln 2$



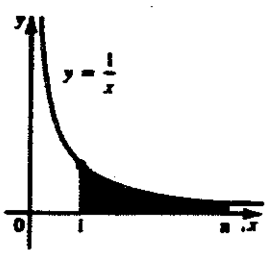
(b) $A(3) = \ln 3$



(c) $A(e) = \ln e = 1$

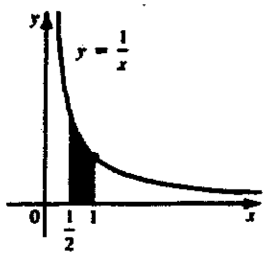


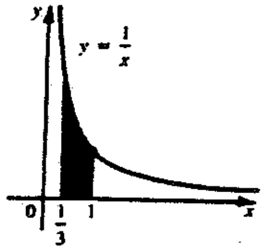
(d) $A(n) = \ln(n)$



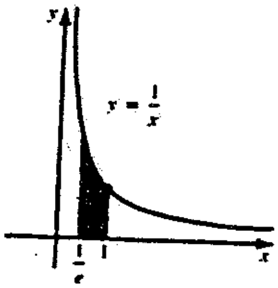
4. $F(x)=\ln x$. $A(1)=F(1)-F(a)=\ln 1-\ln a=-\ln a$.

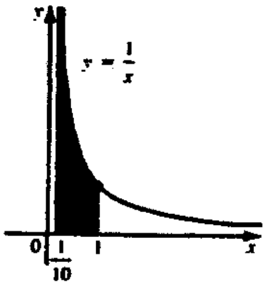
(a) $A(\frac{1}{2})=-\ln\frac{1}{2}=\ln(\frac{1}{2})^{-1}=\ln 2$ (b) $A(\frac{1}{3})=-\ln\frac{1}{3}=\ln 3$



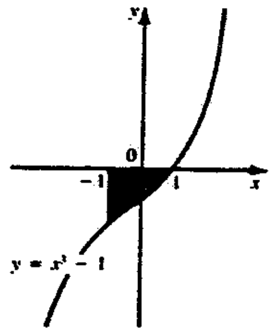


(c) $A(\frac{1}{e})=-\ln\frac{1}{e}=\ln e=1$ (d) $A(\frac{1}{10})=-\ln\frac{1}{10}=\ln 10$

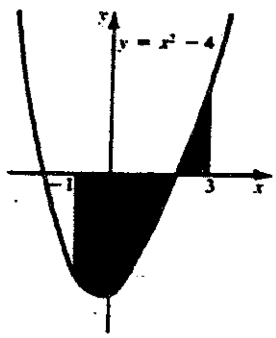




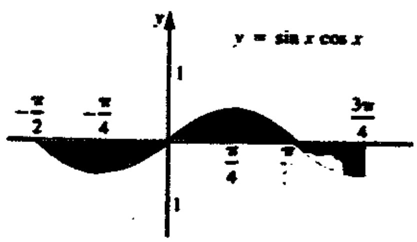
5. A reflection in the x-axis generates an equal area above the x-axis; namely the area under $y=-x^3+1$, from -1 to 1. $F(x)=-\frac{1}{4}x^4+x$. $A(1)=F(1)-F(-1)$ $=-\frac{1}{4}+1+\frac{1}{4}+1=2$



6. $x^2-4=0 \Rightarrow (x+2)(x-2)=0$. The x-intercepts are -2 and 2. The region from -1 to 2 lies below the x-axis. A reflection in the x-axis generates an equal area above the x-axis; namely the area under $y=4-x^2$, from -1 to 2. $F_1(x)=4x-\frac{1}{3}x^3$. $A_1(2)=F_1(2)-F_1(-1)$ $=8-\frac{8}{3}+4-\frac{1}{3}=9$. The region from 2 to 3 is found in the normal way. $F_2(x)=\frac{1}{3}x^3-4x$. $A_2(3)=F_2(3)-F_2(2)$ $=9-12-\frac{8}{3}+8=2\frac{1}{3}$. The required area is $11\frac{1}{3}$.



7. $y=\sin x\cos x=\frac{1}{2}\sin 2x$. The required area is 2.5 times the area under $y=\frac{1}{2}\sin 2x$ from 0 to $\frac{\pi}{2}$.
$F(x)=-\frac{1}{4}\cos 2x$. $A(\frac{\pi}{2})=F(\frac{\pi}{2})-F(0)=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$.
The required area is $2.5\times 0.5=1.25$.



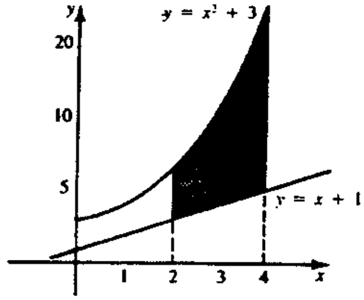
EXERCISE 10.2

1. (a) $A'(x)=x^2+3-x-1=x^2-x+2$

$F(x)=\frac{1}{3}x^3-\frac{1}{2}x^2+2x.$

$A(4)=F(4)-F(2)$

$=\frac{64}{3}-8+8-\frac{8}{3}+2-4=\frac{50}{3}$

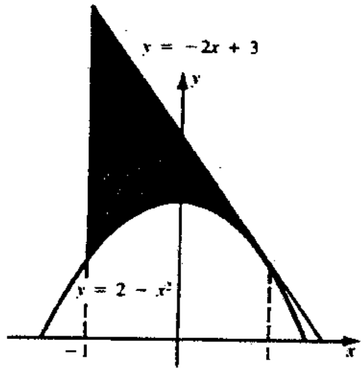


(b) $A'(x)=-2x+3-2+x^2=x^2-2x+1$

$F(x)=\frac{1}{3}x^3-x^2+x.\ A(1)$

$=F(1)-F(-1)$

$=\frac{1}{3}-1+1+\frac{1}{3}+1+1=\frac{8}{3}$



(c) The points of intersection occur when

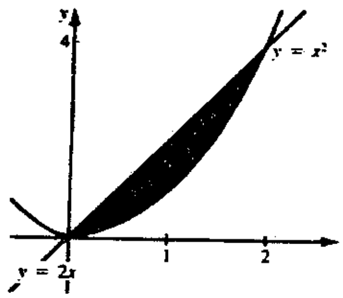
$x^2=2x \Rightarrow x^2-2x=0 \Rightarrow x(x-2)=0$

$\Rightarrow x=0$ or $2.$

$A'(x)=2x-x^2. F(x)=x^2-\frac{1}{3}x^3.$

$A(2)=F(2)-F(0)$

$=4-\frac{8}{3}=\frac{4}{3}$



(d) The points of intersection occur when

$4-x^2=2x+1 \Rightarrow 0=x^2+2x-3$

$\Rightarrow (x+3)(x-1)=0$

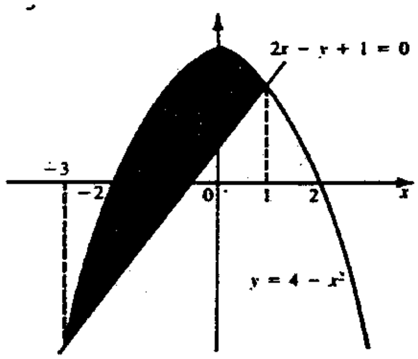
$\Rightarrow x=1$ or $x=-3.\ A'(x)=4-x^2-2x-1$

$\Rightarrow A'(x)=-x^2-2x+3.$

$F(x)=-\frac{1}{3}x^3-x^2+3x$

$A(1)=F(1)-F(-3)$

$=-\frac{1}{3}-1+3-9+9+9=\frac{32}{3}$



(e) The points of intersection occur when

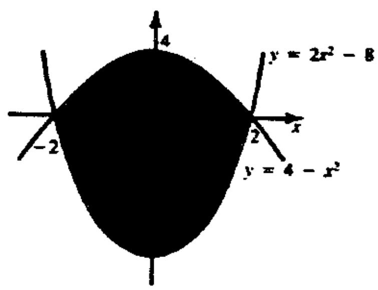
$4-x^2=2x^2-8 \Rightarrow 3x^2=12 \Rightarrow x=\pm 2.$

$A'(x)=4-x^2-2x^2+8=-3x^2+12$

$F(x)=-x^3+12x.\ A(2)$

$=F(2)-F(-2)$

$=-8+24-8+24=32$

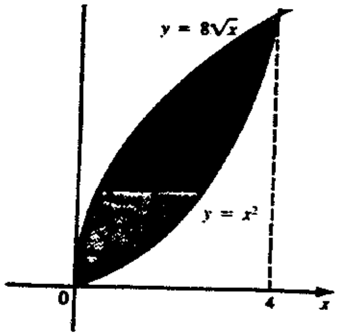


Exercise 10.2

(f) The points of intersection occur when

$x^2 = 8\sqrt{x} \Rightarrow x^4 - 64x = 0 \Rightarrow x(x^3 - 64) = 0$

$\Rightarrow x = 0$ or $x = 4$. $A'(x) = 8\sqrt{x} - x^2$.



$F(x) = \frac{16}{3}\sqrt{x^3} - \frac{1}{3}x^3$.

$A(4) = F(4) - F(0) = \frac{128}{3} - \frac{64}{3} = \frac{64}{3}$

(g) The points of intersection occur when

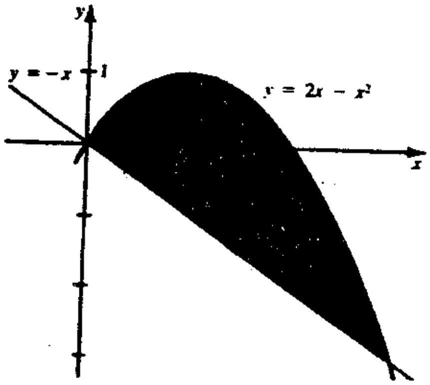
$2x - x^2 = -x \Rightarrow x^2 - 3x = 0 \Rightarrow x(x-3) = 0$

$\Rightarrow x = 0$ or $x = 3$.

$A'(x) = 2x - x^2 + x = 3x - x^2$.

$F(x) = \frac{3}{2}x^2 - \frac{1}{3}x^3$. $A(3) = F(3) - F(0)$

$= \frac{27}{2} - 9 = \frac{9}{2}$

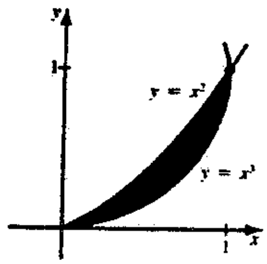


(h) The points of intersection occur when

$x^3 = x^2 \Rightarrow x^2(x-1) = 0 \Rightarrow x = 0$ or $x = 1$.

$A'(x) = x^2 - x^3$. $F(x) = \frac{1}{3}x^3 - \frac{1}{4}x^4$.

$A(1) = F(1) - F(0) = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$.



(i) The points of intersection occur when

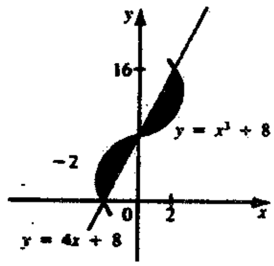
$x^3 + 8 = 4x + 8 \Rightarrow x(x^2 - 4) = 0 \Rightarrow x = -2$,

$x = 0$ or $x = 2$.

$A_2(x) = 4x - x^3$. $F_2(x) = 2x^2 - \frac{x^4}{4}$.

$A_2(2) = F_2(2) - F_2(0) = 8 - 4 = 4$.

$A_1 = A_2$, since both curves are symmetric about the point $(0,2)$. Therefore the required area is $2A_2 = 8$.



(j) The points of intersection occur when

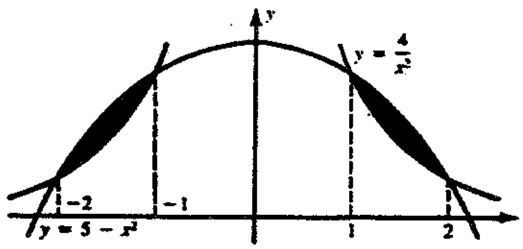
$\frac{4}{x^2} = 5 - x^2 \Rightarrow x^4 - 5x^2 + 4 = 0$

$\Rightarrow (x^2 - 1)(x^2 - 4) = 0$

$\Rightarrow x = \pm 1$ or $x = \pm 2$. $A'(x) = 5 - x^2 - \frac{4}{x^2}$.

$F(x) = 5x - \frac{1}{3}x^3 + \frac{4}{x}$.

$A_2(2) = F_2(2) - F_2(1)$

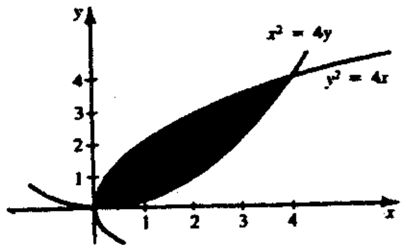


$= 10 - \frac{8}{3} + 2 - 5 + \frac{1}{3} - 4 = \frac{2}{3}.$

$A_1(-1) = A_2(2)$ since both functions are even.

Therefore the area is $\frac{4}{3}$.

(k) The points of intersection occur when



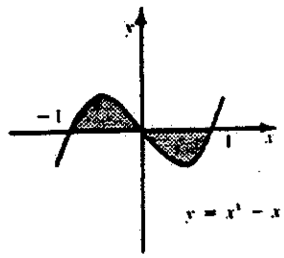
$2\sqrt{x} = \frac{x^2}{4} \Rightarrow 64x = x^4 \Rightarrow x(x^3 - 64) = 0$

$\Rightarrow x = 0$ or $x = 4$. $A'(x) = 2\sqrt{x} - \frac{1}{4}x^2$.

$F(x) = \frac{4}{3}\sqrt{x^3} - \frac{1}{12}x^3$.

$A(4) = F(4) - F(0) = \frac{32}{3} - \frac{16}{3} = \frac{16}{3}.$

(l) The points of intersection occur when



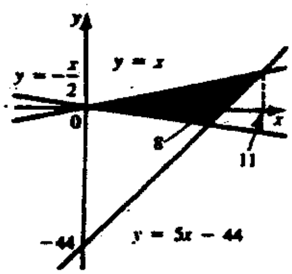
$x^3 - x = 0 \Rightarrow x(x^2 - 1) = 0$

$\Rightarrow x = -1$ or 0 or 1.

$A'(x) = x^3 - x$. $F(x) = \frac{1}{4}x^4 - \frac{1}{2}x^2$.

$A(0) = F(0) - F(-1) = -\frac{1}{4} + \frac{1}{2} = \frac{1}{4}$. Since the function is odd the areas are symmetric and the required area is $2A(0) = \frac{1}{2}$.

(m) The points of intersection occur when



$x = -\frac{1}{2}x$ and $-\frac{1}{2}x = 5x - 44$ and $5x - 44 = x$

$\Rightarrow x = 0$ and $x = 8$ and $x = 11$.

First calculate A_1.

$A_1'(x) = x + \frac{1}{2}x$. $F_1(x) = \frac{3}{4}x^2$.

$A_1(8) = F_1(8) - F_1(0) = 48.$

Now calculate A_2.

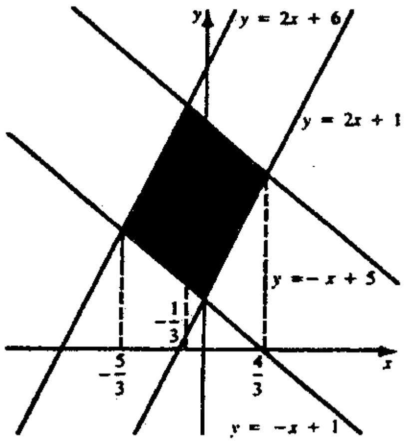
$A_2'(x) = x - 5x + 44 = 44 - 4x.$

$F_2(x) = 44x - 2x^2$. $A_2(11) = F_2(11) - F_2(8)$

$= 484 - 242 - 352 + 128 = 18.$

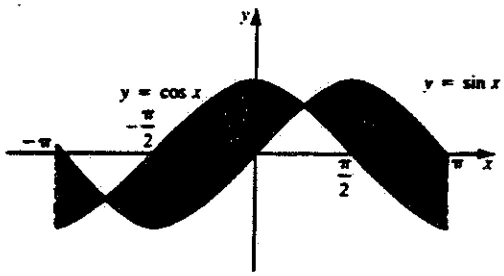
Area $= 18 + 48 = 66.$

(n) The points of intersection occur when $1-x=2x+1$ and $5-x=2x+1$ and $1-x=2x+6$ and $5-x=2x+6$ $\Rightarrow$ $x=0$ and $x=\frac{4}{3}$ and $x=-\frac{5}{3}$ and $x=-\frac{1}{3}$. First calculate A_1. $A_1'(x)=5-x-2x-1$ $=4-3x$. $F_1(x)=4x-\frac{3}{2}x^2$. $A_1(\frac{4}{3})=F_1(\frac{4}{3})-F_1(0)$ $=\frac{16}{3}-\frac{8}{3}=\frac{8}{3}$. Now calculate A_2. $A'_2(x)=5-x+x-1=4$. $F_2(x)=4x$. $A_2(0)=F_2(0)-F_2(-\frac{1}{3})=0+\frac{4}{3}=\frac{4}{3}$. Now calculate A_3. $A_3'(x)=2x+6+x-1=3x+5$. $F_3(x)=\frac{3}{2}x^2+5x$. $A_3(-\frac{1}{3})=F_3(-\frac{1}{3})-F_3(-\frac{5}{3})$ $=\frac{1}{6}-\frac{5}{3}-\frac{25}{6}+\frac{25}{3}=\frac{8}{3}$. Area $=\frac{8}{3}+\frac{4}{3}+\frac{8}{3}=\frac{20}{3}$.



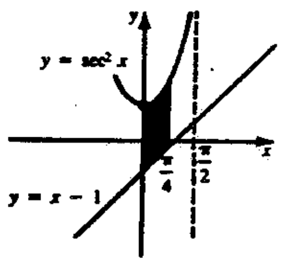
(o) The points of intersection occur when $\sin x=\cos x$ $\Rightarrow$ $\tan x=1$ $\Rightarrow$ $x=-\frac{3}{4}\pi$ and $\frac{\pi}{4}$. Since $A_2=A_4$, the total area is $2A_1$. $A_1'(x)=\cos x-\sin x$. $F_1(x)=\sin x+\cos x$. $A_1(\frac{\pi}{4})=F_1(\frac{\pi}{4})-F_1(-\frac{3}{4}\pi)=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}$

$=\frac{4}{\sqrt{2}}=2\sqrt{2}$. The total area is $4\sqrt{2}$

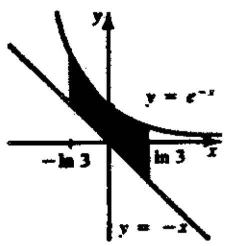


(p) $A'(x)=\sec^2 x-x+1$. $F(x)=\tan x-\frac{1}{2}x^2+x$. $A(\frac{\pi}{4})=F(\frac{\pi}{4})-F(0)$

$=1-\frac{\pi^2}{32}+\frac{\pi}{4}=\frac{32-\pi^2+8\pi}{32}$.



(q) $A'(x)=e^{-x}+x$. $F(x)=-e^{-x}+\frac{1}{2}x^2$. $A(\ln 3)=F(\ln 3)-F(-\ln 3)$ $=-\frac{1}{3}+\frac{1}{2}(\ln 3)^2+3-\frac{1}{2}(\ln 3)^2$ $=\frac{8}{3}$.



(r) The points of intersection occur when

$\frac{1}{x} = 2 - x \Rightarrow x^2 - 2x + 1 = 0$

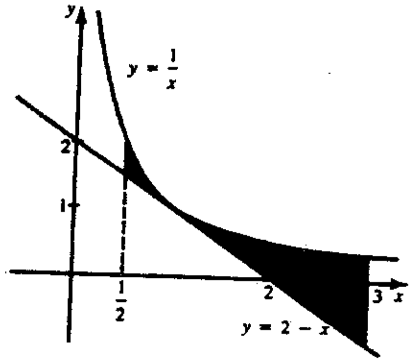
$\Rightarrow (x-1)^2 = 0 \Rightarrow x = 1.$

$A'(x) = \frac{1}{x} - 2 + x.\ F(x) = \ln x - 2x + \frac{1}{2}x^2.$

$A(3) = F(3) - F(\frac{1}{2})$

$= \ln 3 - 6 + \frac{9}{2} - \ln\frac{1}{2} + 1 - \frac{1}{8}$

$= \ln 3 + \ln 2 - \frac{5}{8}.$



(s) The points of intersection occur when

$\sin x = \cos 2x \Rightarrow \sin x = 1 - 2\sin^2 x$

$\Rightarrow 2\sin^2 x + \sin x - 1 = 0$

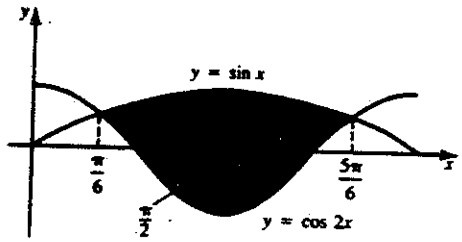
$\Rightarrow (2\sin x - 1)(\sin x + 1) = 0$

$\Rightarrow \sin x = \frac{1}{2}$ or $\sin x = -1 \Rightarrow x = \frac{\pi}{6}$ or $x = \frac{5}{6}\pi.$

$A'(x) = \sin x - \cos 2x.\ F(x) = -\cos x - \frac{1}{2}\sin 2x.$

$A(\frac{5}{6}\pi) = F(\frac{5}{6}\pi) - F(\frac{\pi}{6})$

$= \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{4} = \frac{3\sqrt{3}}{2}.$



(t) The points of intersection occur when

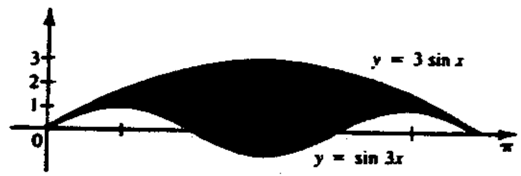
$3\sin x = \sin 3x \Rightarrow 3\sin x = 3\sin x - 4\sin^3 x$

$\Rightarrow 4\sin^3 x = 0 \Rightarrow \sin x = 0 \Rightarrow x = 0$ or $x = \pi.$

$A'(x) = 3\sin x - \sin 3x.$

$F(x) = -3\cos x + \frac{1}{3}\cos 3x.$

$A(\pi) = F(\pi) - F(0) = 3 - \frac{1}{3} + 3 - \frac{1}{3} = \frac{16}{3}.$



2. (a) Consider $y = |x-1| + |x+1|$

If $x \leq -1$, $y = -2x.$

If $-1 < x < 1$, $y = 2.$

If $x \geq 1$, $y = 2x.$

The points of intersection occur when

$-2x = 3 - x^2$ and $2x = 3 - x^2$

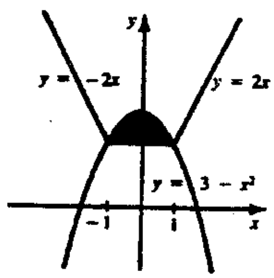
$\Rightarrow x^2 - 2x - 3 = 0$ and $x^2 + 2x - 3 = 0$

$\Rightarrow (x-3)(x+1) = 0$ and $(x+3)(x-1) = 0$

$\Rightarrow x = -1$ and $x = 1.$

$A'(x) = 3 - x^2 - 2 = 1 - x^2.\ F(x) = x - \frac{1}{3}x^3.$

$A(1) = F(1) - F(-1) = 1 - \frac{1}{3} + 1 - \frac{1}{3} = \frac{4}{3}.$



(b) Consider $y = |x| - x$.

If $x \leq 0$, $y = -2x$. If $x > 0$, $y = 0$.

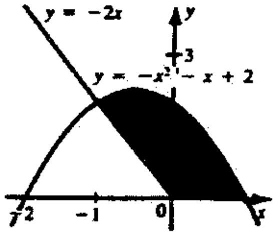
The points of intersection occur when

$-2x = -x^2 - x + 2$ and $0 = -x^2 - x + 2$

$\Rightarrow x^2 - x - 2 = 0$ and $x^2 + x - 2 = 0$

$\Rightarrow (x-2)(x+1) = 0$ and $(x+2)(x-1) = 0$

$\Rightarrow x = -1$ and $x = 1$.



$A_1'(x) = -x^2 - x + 2 + 2x = -x^2 + x + 2$.

$F_1(x) = -\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x$.

$A_1(0) = F_1(0) - F_1(-1)$

$= -\frac{1}{3} - \frac{1}{2} + 2 = 1\frac{1}{6}$.

$A_2'(x) = -x^2 - x + 2$. $F(x) = -\frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x$.

$A_2(1) = F_2(1) - F_2(0) = -\frac{1}{3} - \frac{1}{2} + 2 = \frac{7}{6}$.

Area $= A_1 + A_2 = \frac{7}{3}$.

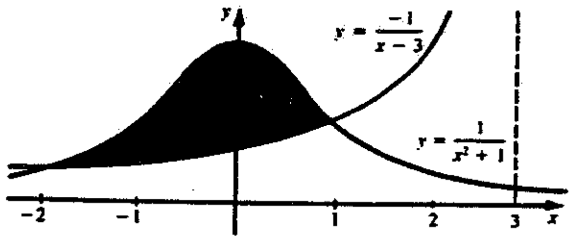
(c) The points of intersection occur when

$\frac{1}{3-x} = \frac{1}{x^2+1} \Rightarrow x^2 + 1 = 3 - x$

$\Rightarrow x^2 + x - 2 = 0$

$\Rightarrow (x+2)(x-1) = 0 \Rightarrow x = -2$ and $x = 1$.

$A'(x) = \frac{1}{x^2+1} - \frac{1}{3-x}$.



$F(x) = \tan^{-1}x + \ln|3 - x|$.

$A(1) = F(1) - F(-2)$

$= \frac{\pi}{4} + \ln 2 - \tan^{-1}(-2) - \ln|5|$

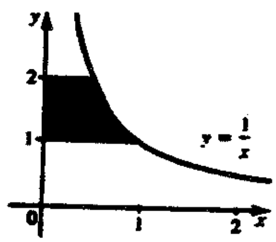
$= \frac{\pi}{4} + \tan^{-1}2 + \ln\frac{2}{5}$.

3. (a) Rewrite $y = \frac{1}{x}$ as a function of y;

namely $x = \frac{1}{y}$. $A'(y) = \frac{1}{y}$. $F(y) = \ln y$.

$A(2) = F(2) - F(1)$

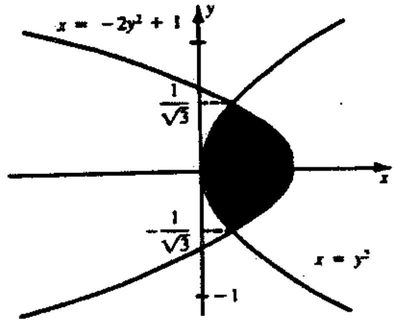
$= \ln 2 - \ln 1 = \ln 2$.



(b) Treat both curves as functions of y.

The points of intersection occur when

$y^2 = -2y^2 + 1 \Rightarrow 3y^2 = 1 \Rightarrow y = \pm\frac{1}{\sqrt{3}}$



$A'(y) = -2y^2 + 1 - y^2 = -3y^2 + 1.$

$F(y) = -y^3 + y$. We calculate the area from $y = 0$ to $y = \frac{1}{\sqrt{3}}$ and double our result.

$A(\frac{1}{\sqrt{3}}) = F(\frac{1}{\sqrt{3}}) - F(0) = -\frac{1}{3\sqrt{3}} + \frac{1}{\sqrt{3}} = \frac{2}{3\sqrt{3}}$

The required area is $\frac{4}{3\sqrt{3}} = \frac{4\sqrt{3}}{9}$.

(c) Rewrite both curves as functions of y;

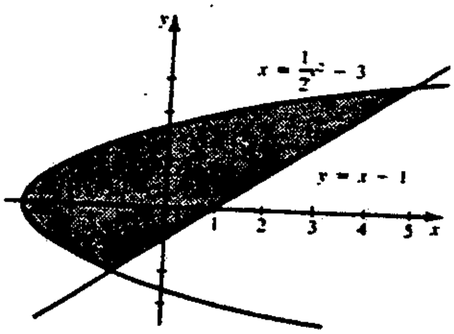
namely, $x = y + 1$ and $x = \frac{y^2 - 6}{2}$. The points of intersection occur when $2y + 2 = y^2 - 6$

$\Rightarrow y^2 - 2y - 8 = 0 \Rightarrow (y - 4)(y + 2) = 0$

$\Rightarrow y = -2$ and $y = 4$. $A'(y) = y + 1 - \frac{1}{2}y^2 + 3$

$\Rightarrow A'(y) = -\frac{1}{2}y^2 + y + 4.$

$F(y) = -\frac{1}{6}y^3 + \frac{1}{2}y^2 + 4y.$



$A(4) = F(4) - F(-2)$

$= -\frac{64}{6} + 8 + 16 - \frac{8}{6} - 2 + 8 = 18.$

EXERCISE 10.3

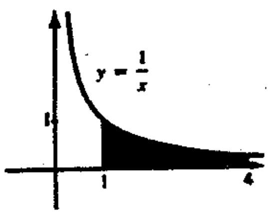
1. (a) $A = \ln 3 - \ln 2 = \ln 1.5 \doteq 0.405465$

   (b) $A = -\ln \frac{1}{4} - (-\ln \frac{1}{2}) = \ln \frac{1}{2} - \ln \frac{1}{4} = \ln 2 = 0.693147$

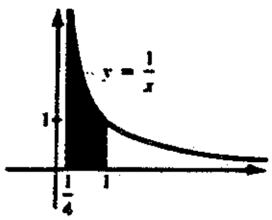
   (c) $A = \ln 2 - \ln \frac{2}{3} = \ln 3 = 1.098612$

   (d) $A = 2(\ln 3 - \ln 1.5) = 2\ln 2 = \ln 4 = 1.386294$

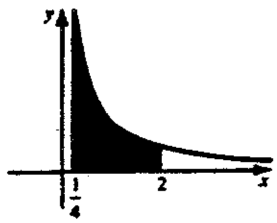
2. (a) $A = \ln 4$



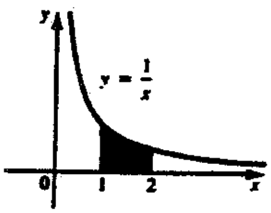
(b) $A = -\ln \frac{1}{4}$



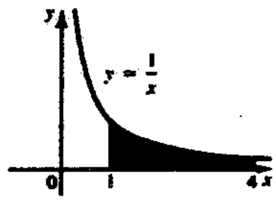
(c) $\ln 2 + \ln 4 = \ln 8$



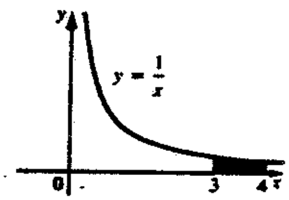
(d) $\ln 6 - \ln 3 = \ln 2$



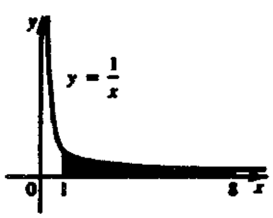
(e) $2\ln 2 = \ln 4$



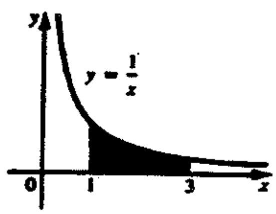
(f) $-\ln \frac{3}{4} = \ln \frac{4}{3} = \ln 4 - \ln 3$



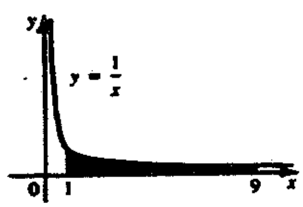
(g) $-\ln \frac{1}{2} - \ln \frac{1}{4}$
$= \ln 2 + \ln 4 = \ln 8$



(h) $-\frac{1}{2}\ln \frac{1}{9} = -\ln \frac{1}{3}$
$= \ln 3$



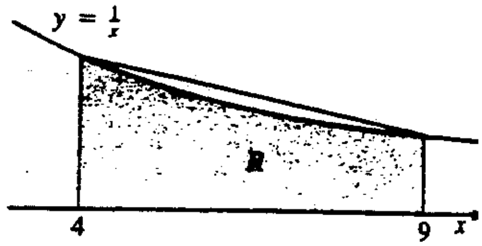
(i) $-\ln \frac{1}{3} + \ln 3$
$= \ln 3 + \ln 3 = \ln 9$



3. $A = \ln 2 = \ln \frac{3}{1.5} = \ln 3 - \ln 1.5 = A_1 - A_2$.

4. $A_1 = \ln 4 - \ln 2 = \ln 2 = \ln(\frac{1}{2})^{-1} = -\ln(\frac{1}{2}) = A_2$.

5. (a) The slanted side of the trapezoid meets $y=\frac{1}{x}$ at $A(4,\frac{1}{4})$ and $B(9,\frac{1}{9})$. The slope of the tangent is the slope of $AB=-\frac{1}{36}$. The slope of the tangent to a point on $y=\frac{1}{x}$ is $y'=-\frac{1}{x^2}$. Setting $x^2=36$

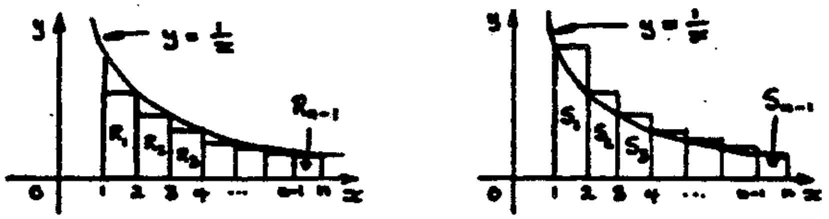


gives us the particular point $(6,\frac{1}{6})$.
The equation of the tangent is $y-\frac{1}{6}=-\frac{1}{36}(x-6)$
$\Rightarrow 36y-6=-x+6 \Rightarrow x+36y-12=0.$
(b) $x+36y-12=0$ meets $x=4$ when $y=\frac{2}{9}$ and $x=9$ when $y=\frac{1}{12}$. Now the area under $y=\frac{1}{x}$ from 4 to 9 is greater than the area of the trapezoid with the tangent as its slanted side, therefore $\ln 9-\ln 4>\frac{1}{2}(9-4)(\frac{2}{9}+\frac{1}{12})$
$\Rightarrow \ln\frac{9}{4}>\frac{5}{2}\times\frac{11}{36} \Rightarrow \ln(\frac{3}{2})^2>\frac{55}{72} \Rightarrow 2\ln 1.5>\frac{55}{72} \Rightarrow \ln 1.5>\frac{55}{144}$

6. (a) R is the region under $y=\frac{1}{x}$ from 3 to 6.
The area of R is $\ln 6-\ln 3=\ln\frac{6}{3}=\ln 2$
(b) The area of R is less than the area of the trapezoid with vertices $(3,0)$, $(6,0)$, $(6,\frac{1}{6})$, and $(3,\frac{1}{3})$, therefore $\ln 2<\frac{1}{2}(6-3)(\frac{1}{3}+\frac{1}{6}) \Rightarrow \ln 2<\frac{3}{2}\times\frac{1}{2} \Rightarrow \ln 2<\frac{3}{4}$

7. (a) R is the region under $y=\frac{1}{x}$ from 18 to 36. The area of R is $\ln 36-\ln 18=\ln\frac{36}{18}=\ln 2$
(b) The area of R is less than the area of the trapezoid with vertices $(18,0)$, $(36,0)$, $(36,\frac{1}{36})$, and $(18,\frac{1}{18})$, therefore $\ln 2<\frac{1}{2}(36-18)(\frac{1}{18}+\frac{1}{36})$
$\Rightarrow \ln 2<9\times\frac{1}{12} \Rightarrow \ln 2<\frac{3}{4}$

8. (a)



The area of R_i is $\frac{1}{1+i}$, therefore $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots+\frac{1}{n}$ is the sum of the areas of $n-1$ rectangles with total area less than the area under $y=\frac{1}{x}$ from 1 to n, namely $\ln n$. The area of S_i is $\frac{1}{i}$, therefore $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots+\frac{1}{n-1}$ is the sum of the areas of $n-1$ rectangles with total area greater than the area under $y=\frac{1}{x}$ from 1 to n. Hence $\ln n$ lies between the the given series.

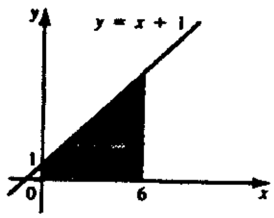
(b) $\ln 2$ is less than the area of the trapezoid with vertices $(1, 0)$, $(2,0)$, $(2,\frac{1}{2})$ and $(1,1)$, therefore $\ln 2 < \frac{1}{2}(2-1)(1+\frac{1}{2}) \Rightarrow \ln 2 < \frac{3}{4} \Rightarrow \ln 2 < 1$.
$\ln 3$ is greater than the area of the trapezoid with slanted side the tangent to $y = \frac{1}{x}$ at $x = 2$. Since $y' = -\frac{1}{x^2}$ the slope of this tangent is $-\frac{1}{4}$ and

its equation is $x + 4y = 4$. It meets $x = 1$ when $y = \frac{3}{4}$ and $x = 3$ when $y = \frac{1}{4}$.
$\ln 3$ is greater than the area of the trapezoid with vertices $(1,0)$, $(3,0)$, $(3,\frac{1}{4})$ and $(1,\frac{3}{4})$, therefore $\ln 3 > \frac{1}{2}(3-1)(\frac{1}{4}+\frac{3}{4}) \Rightarrow \ln 3 > 1$.

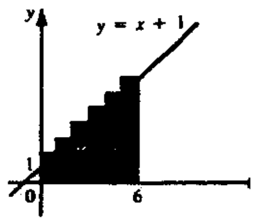
EXERCISE 10.4

1. (a) $A=\frac{1}{2}(6-0)[f(6)+f(0)]=3(7+1)=24$

(a)



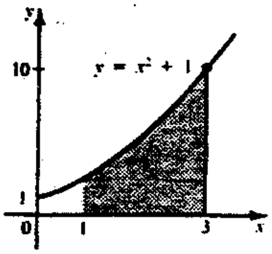
(b)

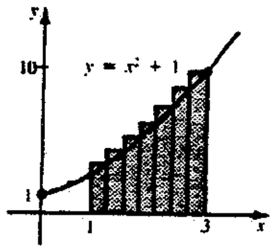


(b) $A \doteq \sum_{i=1}^{6} 1\,f(i)=1(2+3+4+5+6+7)=28$

2. (a) $A'(x)=x^2+1 \Rightarrow F(x)=\frac{1}{3}x^3+x.$

$A(3) = F(3)-F(1)=9+3-\frac{1}{3}-1=\frac{32}{3}$



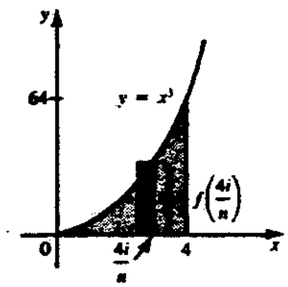


(b) $A \doteq \frac{3-1}{10}\sum_{i=1}^{10} f\left[1+\frac{i}{5}\right]^2$

$=\frac{1}{5}\left\{\left[1+\left(\frac{6}{5}\right)^2\right]+\left[1+\left(\frac{7}{5}\right)^2\right]+\cdots+\left[1+\left(\frac{15}{5}\right)^2\right]\right\}$

$=\frac{1}{5}\left[10+\frac{1}{25}(6^2+7^2+8^2+\cdots+15^2)\right]=2+\frac{1185}{125}=\frac{1435}{125}=11.48$

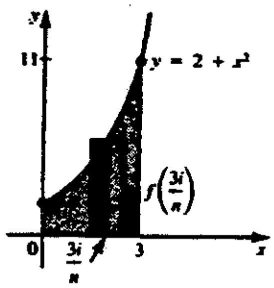
3. (a) $\Delta x=\frac{4}{n};\ x_i=\frac{4i}{n};\ \sum_{i=1}^{n}\left[\frac{4}{n}f\left(\frac{4i}{n}\right)\right]$



$=\frac{4}{n}\sum_{i=1}^{n}\frac{64i^3}{n^3}=\frac{256}{n^4}\left[\frac{n^2(n+1)^2}{4}\right]=64\left(1+\frac{1}{n}\right)^2$

$A=64\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^2=64$

(b) $\Delta x=\frac{3}{n};\ x_i=\frac{3i}{n};\ \sum_{i=1}^{n}\left[\frac{3}{n}f\left(\frac{3i}{n}\right)\right]=\frac{3}{n}$

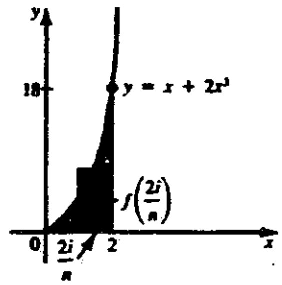


$\sum_{i=1}^{n}\left[2+\frac{9i^2}{n^2}\right]=\frac{3}{n}\left[2n+\frac{9}{n^2}\times\frac{n(n+1)(2n+1)}{6}\right]$

$A=\lim_{n\to\infty}\left[6+\frac{9}{2}(1)\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)\right]=6+\frac{9}{2}\times1\times1\times2=15$

Exercise 10.4

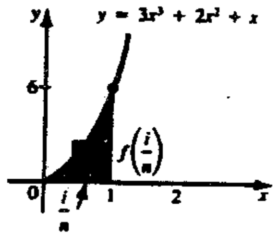
(c) $\Delta x = \frac{2}{n};\ x_i = \frac{2i}{n};\quad \sum_{i=1}^{n}\left[\frac{2}{n}f\left(\frac{2i}{n}\right)\right] = \frac{2}{n}\sum_{i=1}^{n}\left[\frac{2i}{n} + \frac{16i^3}{n^3}\right]$



$$= \frac{4}{n^2}\times\frac{n(n+1)}{2} + \frac{32}{n^4}\times\frac{n^2(n+1)^2}{4}.$$

$$A = 2\lim_{n\to\infty}\left(1+\frac{1}{n}\right) + 8\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^2 = 2 + 8 = 10$$

(d) $\Delta x = \frac{1}{n};\ x_i = \frac{i}{n};\quad \sum_{i=1}^{n}\left[\frac{1}{n}f\left(\frac{i}{n}\right)\right] = \frac{1}{n}\sum_{i=1}^{n}\left[\frac{3i^3}{n^3} + \frac{2i^2}{n^2} + \frac{i}{n}\right]$

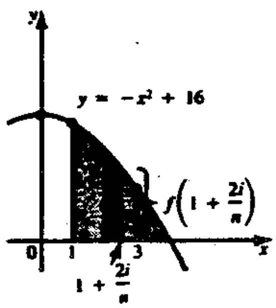


$$= \frac{3}{n^4}\times\frac{n^2(n+1)^2}{4} + \frac{2}{n^3}\times\frac{n(n+1)(2n+1)}{6}\times\frac{1}{n^2}\times\frac{n(n+1)}{2}$$

$$A = \frac{3}{4}\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^2 + \frac{1}{3}\lim_{n\to\infty}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) + \frac{1}{2}\lim_{n\to\infty}\left(1+\frac{1}{n}\right)$$

$$= \frac{3}{4} + \frac{2}{3} + \frac{1}{2} = \frac{23}{12}$$

4. (a) $\Delta x = \frac{3-1}{n} = \frac{2}{n};\ x_i = 1 + \frac{2i}{n};$



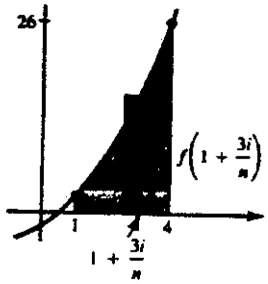
$$\sum_{i=1}^{n}\left[\frac{2}{n}f\left(1+\frac{2i}{n}\right)\right] = \frac{2}{n}\sum_{i=1}^{n}\left[-\left(1+\frac{2i}{n}\right)^2 + 16\right]$$

$$= \frac{2}{n}\sum_{i=1}^{n}\left[15 - \frac{4i}{n} - \frac{4i^2}{n^2}\right]$$

$$= \frac{2}{n}\left[15n - \frac{4n(n+1)}{2n} - \frac{4n(n+1)(2n+1)}{6n^2}\right]$$

$$A = \lim_{n\to\infty}\left[30 - 4\left(1+\frac{1}{n}\right) - \frac{4}{3}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)\right] = 30 - 4 - \frac{8}{3} = \frac{70}{3}$$

Exercise 10.4

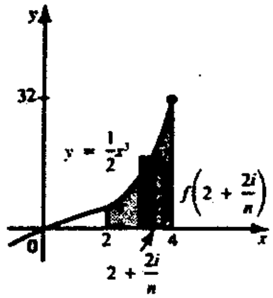


(b) $\Delta x = \frac{3}{n};\ x_i = 1 + \frac{3i}{n};$

$$\sum_{i=1}^{n}\left[\frac{3}{n}f\left(1+\frac{3i}{n}\right)\right] = \frac{3}{n}\sum_{i=1}^{n}\left[\left(1+\frac{3i}{n}\right)^2 + 3\left(1+\frac{3i}{n}\right) - 2\right]$$

$$= \frac{3}{n}\sum_{i=1}^{n}\left[2 + \frac{15i}{n} + \frac{9i^2}{n^2}\right] = \frac{3}{n}\left[2n + \frac{15n(n+1)}{2n} + \frac{9n(n+1)(2n+1)}{6n^2}\right]$$

$$A = \lim_{n\to\infty}\left[6 + \frac{45}{2}\left(1+\frac{1}{n}\right) + \frac{9}{2}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)\right] = 6 + \frac{45}{2} + 9 = \frac{75}{2}$$



(c) $\Delta x = \frac{2}{n};\ x_i = 2 + \frac{2i}{n};$

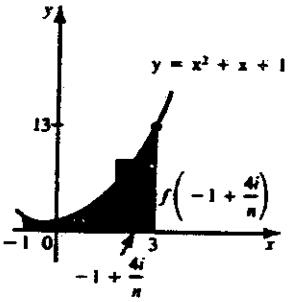
$$\sum_{i=1}^{n}\left[\frac{2}{n}f\left(2+\frac{2i}{n}\right)\right] = \frac{2}{n}\sum_{i=1}^{n}\left[\frac{1}{2}\left(2+\frac{2i}{n}\right)^3\right]$$

$$= \frac{8}{n}\sum_{i=1}^{n}\left[1 + \frac{3i}{n} + \frac{3i^2}{n^2} + \frac{i^3}{n^3}\right]$$

$$= \frac{8}{n}\left[n + \frac{3n(n+1)}{2n} + \frac{3n(n+1)(2n+1)}{6n^2} + \frac{n^2(n+1)^2}{4n^3}\right]$$

$$A = \lim_{n\to\infty}\left[8 + 12\left(1+\frac{1}{n}\right) + 4\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) + 2\left(1+\frac{1}{n}\right)^2\right]$$

$$= 8 + 12 + 8 + 2 = 30$$



(d) $\Delta x = \frac{4}{n};\ x_i = -1 + \frac{4i}{n};$

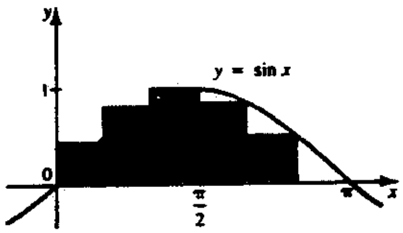
$$\sum_{i=1}^{n}\left[\frac{4}{n}f\left(-1+\frac{4i}{n}\right)\right] = \frac{4}{n}\sum_{i=1}^{n}\left[\left(-1+\frac{4i}{n}\right)^2 + \left(-1+\frac{4i}{n}\right) + 1\right]$$

$$= \frac{4}{n}\sum_{i=1}^{n}\left[1 - \frac{4i}{n} + \frac{16i^2}{n^2}\right] = \frac{4}{n}\left[n - \frac{4n(n+1)}{2n} + \frac{16n(n+1)(2n+1)}{6n^2}\right]$$

$$A = \lim_{n\to\infty}\left[4 - 8\left(1+\frac{1}{n}\right) + \frac{32}{3}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)\right] = 4 - 8 + \frac{64}{3} = \frac{52}{3}$$

Exercise 10.4

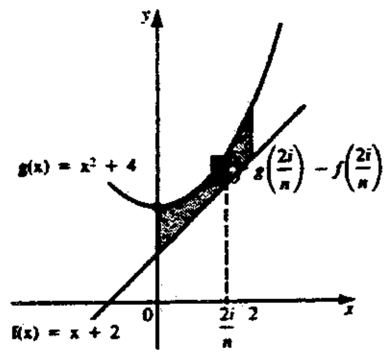
5. $\Delta x = \frac{\pi}{6};\ x_i = \frac{\pi i}{6};\ \sum_{i=1}^{6} \left[\frac{\pi}{6} f\left(\frac{\pi i}{6}\right) \right] = \frac{\pi}{6} \sum_{i=1}^{6} \sin\left(\frac{\pi i}{6}\right)$



$$A = \frac{\pi}{6}\left(\sin\frac{\pi}{6} + \sin\frac{2\pi}{6} + \sin\frac{3\pi}{6} + \sin\frac{4\pi}{6} + \sin\frac{5\pi}{6} + \sin\pi \right)$$

$$= \frac{\pi}{6}\left(\frac{1}{2} + \frac{\sqrt{3}}{2} + 0 + \frac{\sqrt{3}}{2} + \frac{1}{2} + 0\right) = \frac{\pi}{6}(1 + \sqrt{3})$$

6. (a) $\Delta x = \frac{2}{n};\ x_i = \frac{2i}{n};\ \sum_{i=1}^{n} \frac{2}{n}\left[\left(\frac{2i}{n}\right)^2 + 4 - \left(\frac{2i}{n} + 2\right) \right]$



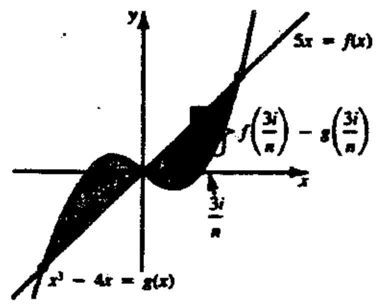
$$= \frac{2}{n} \sum_{i=1}^{n} \left[\frac{4i^2}{n^2} - \frac{2i}{n} + 2 \right]$$

$$= \frac{2}{n}\left[\frac{4n(n+1)(2n+1)}{6n^2} - \frac{2n(n+1)}{2n} + 2n \right]$$

$$A = \lim_{n \to \infty} \left[\frac{4}{3}\left(1 + \frac{1}{n}\right)\left(2 + \frac{1}{n}\right) - 2\left(1 + \frac{1}{n}\right) + 4 \right] = \frac{8}{3} - 2 + 4 = \frac{14}{3}$$

(b) Points of intersection occur when $x^3 - 4x = 5x$
$\Rightarrow x^3 - 9x = 0 \Rightarrow x(x-3)(x+3) = 0 \Rightarrow x = -3, 0, 3.$
Symmetry allows us to calculate the area from 0 to 3 and double our answer for the required area. $\Delta x = \frac{3}{n};\ x_i = \frac{3i}{n};$

$$\sum_{i=1}^{n} \left[\frac{3}{n}\left(\frac{15i}{n} - \frac{27i^3}{n^3} + \frac{12i}{n} \right) \right] = \frac{3}{n} \sum_{i=1}^{n} \left[\frac{27i}{n} - \frac{27i^3}{n^3} \right]$$



$$= \frac{3}{n}\left[\frac{27n(n+1)}{2n} - \frac{27n^2(n+1)^2}{4n^3} \right]$$

$$A = \lim_{n \to \infty} \left[\frac{81}{2}\left(1 + \frac{1}{n}\right) - \frac{81}{4}\left(1 + \frac{1}{n}\right)^2 \right] = \frac{81}{2} - \frac{81}{4} = \frac{81}{4}$$

Therefore the required area is $\frac{81}{2}$.

EXERCISE 10.5

1. (a) (i) $A \doteq \frac{1}{2}\left[\frac{1}{2}\left(1+e^{\frac{1}{2}}\right)+\frac{1}{2}\left(e^{\frac{1}{2}}+e\right)+\frac{1}{2}\left(e+e^{\frac{3}{2}}\right)+\frac{1}{2}\left(e^{\frac{3}{2}}+e^{2}\right)\right]$

$$= \frac{1}{4}\left(1+2e^{\frac{1}{2}}+2e+2e^{\frac{3}{2}}+e^{2}\right) \doteq 6.521\,610$$

(a) (ii) $A \doteq \frac{1}{2}e^{\frac{1}{4}}+\frac{1}{2}e^{\frac{3}{4}}+\frac{1}{2}e^{\frac{5}{4}}+\frac{1}{2}e^{\frac{7}{4}} = \frac{1}{2}\left(e^{\frac{1}{4}}+e^{\frac{3}{4}}+e^{\frac{5}{4}}+e^{\frac{7}{4}}\right) \doteq 6.322\,986$

(b) $A'(x)=e^{x}$. $F(x)=e^{x}$. $A(2)=F(2)+F(0)=e^{2}-1 \doteq 6.389\,056$.

The area of the trapezoids is about 0.132553 too large.

The area of the rectangles is about 0.066070 too small.

2. (a) (i) the area is approximately

$$\frac{1}{2}\times\frac{3\pi}{16}\left[\left(\sin\frac{4\pi}{16}+\sin\frac{7\pi}{16}\right)+\left(\sin\frac{7\pi}{16}+\sin\frac{10\pi}{16}\right)+\left(\sin\frac{10\pi}{16}+\sin\frac{13\pi}{16}\right)+\left(\sin\frac{13\pi}{16}+\sin\pi\right)\right]$$

$$= \frac{3\pi}{32}\left(\sin\frac{4\pi}{16}+2\sin\frac{7\pi}{16}+2\sin\frac{10\pi}{16}+2\sin\frac{13\pi}{16}+\sin\pi\right) \doteq 1.657\,458$$

(a) (ii) $A \doteq \frac{3}{16}\left[\sin\frac{11\pi}{32}+\sin\frac{17\pi}{32}+\sin\frac{23\pi}{32}+\sin\frac{29\pi}{32}\right] \doteq 1.732\,039$

(b) $A'(x)=\sin x$. $F(x)=-\cos x$. $A(\pi)=F(\pi)-F(\frac{\pi}{4})$

$= -\cos\pi+\cos\frac{\pi}{4} = 1.707\,107$

The area of the trapezoids is 0.049649 too small.

The area of the rectangles is 0.024932 too large.

3. $A \doteq \frac{1}{2}\times\frac{1}{3}\left[\left(e+\frac{4}{3}e^{\frac{4}{3}}\right)+\left(\frac{4}{3}e^{\frac{4}{3}}+\frac{5}{3}e^{\frac{5}{3}}\right)+\left(\frac{5}{3}e^{\frac{5}{3}}+2e^{2}\right)+\left(2e^{2}+\frac{7}{3}e^{\frac{7}{3}}\right)\right]$

$$+\frac{1}{2}\times\frac{1}{3}\left[\left(\frac{7}{3}e^{\frac{7}{3}}+\frac{8}{3}e^{\frac{8}{3}}\right)+\left(\frac{8}{3}e^{\frac{8}{3}}+3e^{3}\right)\right]$$

$$= \frac{1}{6}\left[e+\frac{8}{3}e^{\frac{4}{3}}+\frac{10}{3}e^{\frac{5}{3}}+4e^{2}+\frac{14}{3}e^{\frac{7}{3}}+\frac{16}{3}e^{\frac{8}{3}}+3e^{3}\right] \doteq 40.862\,771$$

Exercise 10.5

4. $A \doteq \frac{\pi}{12}\left[\tan\frac{\pi}{24} + \tan\frac{\pi}{8} + \tan\frac{5\pi}{24}\right] \doteq 0.343793$

5. $\ln 3$ is the area under $y = \frac{1}{x}$ from 1 to 3.

$$A \doteq \frac{1}{2} \times \frac{1}{2}\left[\left(1 + \frac{2}{3}\right) + \left(\frac{2}{3} + \frac{1}{2}\right) + \left(\frac{1}{2} + \frac{2}{5}\right) + \left(\frac{2}{5} + \frac{1}{3}\right)\right] = \frac{1}{4}\left(2 + \frac{5}{3} + \frac{4}{5}\right) = 1.116667$$

6. $\frac{b-a}{n} = \frac{2-1}{12} = \frac{1}{12};\ x_i = 1 + \frac{1}{12}i;$

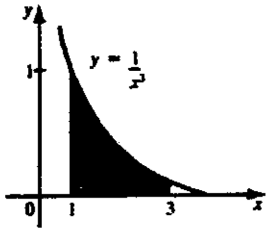
$$A \doteq \frac{1}{2} \times \frac{1}{12}\left[f(1) + 2f\left(\tfrac{13}{12}\right) + 2f\left(\tfrac{14}{12}\right) + 2f\left(\tfrac{15}{12}\right) + \cdots + 2f\left(\tfrac{23}{12}\right) + f(2)\right]$$

$$= \frac{1}{24}\left[\frac{1}{e} + \frac{26}{12e^{\frac{13}{12}}} + \frac{28}{12e^{\frac{14}{12}}} + \frac{30}{12e^{\frac{15}{12}}} + \cdots + \frac{46}{12e^{\frac{23}{12}}} + \frac{2}{e^2}\right] = 0.329675$$

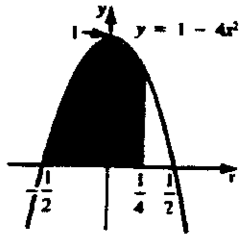
REVIEW EXERCISE 10.6

1. (a) $A'(x) = \frac{1}{x^3}$. $F(x) = -\frac{1}{2x^2}$.

$A(3) = F(3) - F(1) = -\frac{1}{18} + \frac{1}{2} = \frac{4}{9}$.

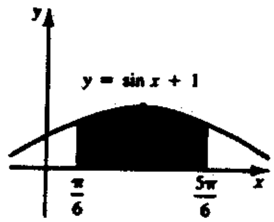


(b) $A'(x) = 1 - 4x^2$. $F(x) = x - \frac{4}{3}x^3$.
$A(\frac{1}{4}) = F(\frac{1}{4}) - F(-\frac{1}{2}) = \frac{1}{4} - \frac{1}{48} + \frac{1}{2} - \frac{1}{6} = \frac{9}{16}$.



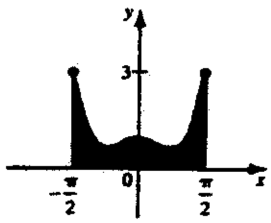
(c) $A'(x) = 1 + \sin x$. $F(x) = x - \cos x$.
$A(\frac{5\pi}{6}) = F(\frac{5\pi}{6}) - F(\frac{\pi}{6})$

$= \frac{5\pi}{6} + \frac{\sqrt{3}}{2} - \frac{\pi}{6} + \frac{\sqrt{3}}{2} = \frac{2\pi}{3} + \sqrt{3}$.

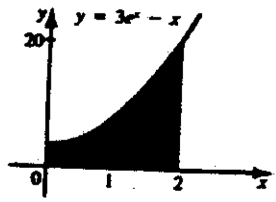


(d) $A'(x) = x^2 + \cos(\frac{1}{2}x)$. $F(x) = \frac{1}{3}x^3 + 2\sin(\frac{1}{2}x)$.
$A(\frac{\pi}{2}) = F(\frac{\pi}{2}) - F(-\frac{\pi}{2})$

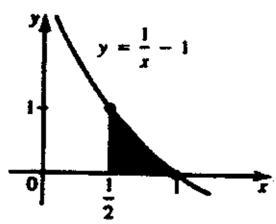
$= \frac{\pi^3}{24} + \frac{2}{\sqrt{2}} + \frac{\pi^3}{24} + \frac{2}{\sqrt{2}} = \frac{\pi^3}{12} + 2\sqrt{2}$.



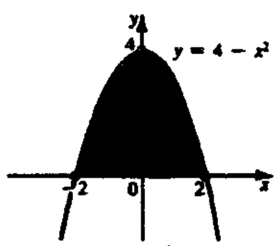
(e) $A'(x) = 3e^x - x$. $F(x) = 3e^x - \frac{1}{2}x^2$.
$A(2) = F(2) - F(0) = 3e^2 - 2 - 3 = 3e^2 - 5$.



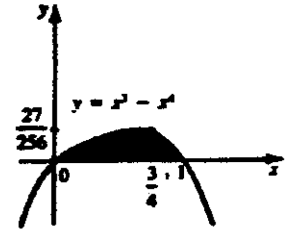
(f) $A'(x) = \frac{1}{x} - 1$. $F(x) = \ln x - x$.
$A(1) = F(1) - F(\frac{1}{2}) = \ln 1 - 1 - \ln\frac{1}{2} + \frac{1}{2} = \ln 2 - \frac{1}{2}$.

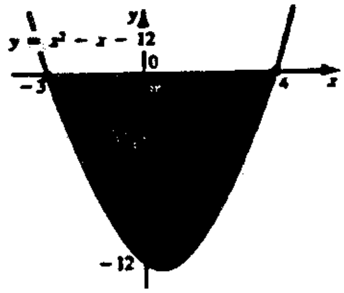


2. (a) $4 - x^2 = 0 \Rightarrow$ x- intercepts are ± 2.
$A'(x) = 4 - x^2$. $F(x) = 4x - \frac{1}{3}x^3$.
We note the symmetry and calculate the area
from 0 to 2 and double our answer.
$A(2) = F(2) - F(0) = 8 - \frac{8}{3} = \frac{16}{3}$. Therefore the required area is $\frac{32}{3}$.



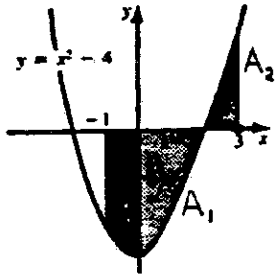
(b) $x^3 - x^4 = 0 \Rightarrow x^3(1-x) = 0 \Rightarrow$ x-intercepts 0, 1.
$A'(x) = x^3 - x^4$. $F(x) = \frac{1}{4}x^4 - \frac{1}{5}x^5$
$A(1) = F(1) - F(0) = \frac{1}{4} - \frac{1}{5} = \frac{1}{20}$.

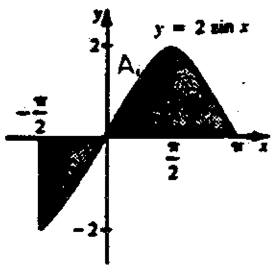




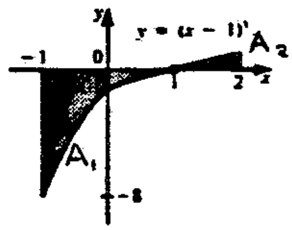
(c) $x^2 - x - 12 = 0 \Rightarrow (x-4)(x+3) = 0 \Rightarrow$ x-intercepts $-3, 4$.
Since the region is below the x-axis,
$A'(x) = -x^2 + x + 12$. $F(x) = -\frac{1}{3}x^3 + \frac{1}{2}x^2 + 12x$.
$A(4) = F(4) - F(-3) = -\frac{64}{3} + 8 + 48 - 9 - \frac{9}{2} + 36 = \frac{343}{6}$.

3. (a) Since $y = x^2 - 4$ crosses the x-axis in the interval $[-1, 3]$ we have a region A_1 below, and a region A_2 above, the x-axis.
$A_1' = 4 - x^2$. $F_1(x) = 4x - \frac{1}{3}x^3$.
$A_1(2) = F_1(2) - F_1(-1) = 8 - \frac{8}{3} + 4 - \frac{1}{3} = 9$.
$A_2'(x) = x^2 - 4$. $F_2(x) = \frac{1}{3}x^3 - 4x$.
$A_2(3) = F_2(3) - F_2(2) = 9 - 12 - \frac{8}{3} + 8 = \frac{7}{3}$.
Therefore $A = \frac{34}{3}$.

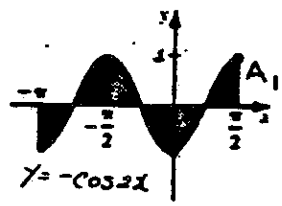




(b) In the interval $[-\frac{\pi}{2}, 0]$ the region is below the x-axis. However, symmetry enables us to find the total area by tripling A_1 the area determined by the interval $[0, \frac{\pi}{2}]$.
$A_1'(x) = 2\sin x$. $F_1(x) = -2\cos x$. $A_1(\frac{\pi}{2}) = F_1(\frac{\pi}{2}) - F_1(0)$ $= 2$. Therefore $A = 3(2) = 6$.

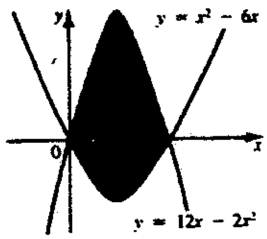


(c) The curve crosses the x-axis at $x = 1$ and two separate regions are determined; A_1 from -1 to 1 and A_2 from 1 to 2. $A_1'(x) = -(x-1)^3$. $F_1(x) = -\frac{1}{4}(x-1)^4$.
$A_1(1) = F_1(1) - F_1(-1) = 4$. $A_2(x) = (x-1)^3$. $F_2(x) = \frac{1}{4}(x-1)^4$
$A_2(2) = F_2(2) - F_2(1) = \frac{1}{4}$. $A = \frac{17}{4}$.

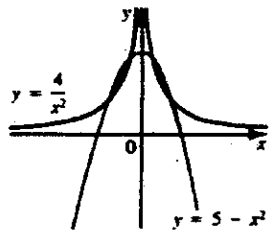


(d) Four distinct regions are determined, two above and two below the x-axis. The larger regions are equal. They are twice the size of the smaller regions. Six times A_1 the region determined by the interval $[\frac{\pi}{4}, \frac{\pi}{2}]$ gives us the total area. $A_1'(x) = -\cos 2x$.
$F_1(x) = -\frac{1}{2}\sin 2x$. $A_1(\frac{\pi}{2}) = F_1(\frac{\pi}{2}) - F_1(\frac{\pi}{4}) = 0 + \frac{1}{2} = \frac{1}{2}$. $A = 3$.

4. (a) The points of intersection occur when $x^2 - 6x = 12x - 2x^2$ $\Rightarrow 3x(x-6) = 0 \Rightarrow x = 0$ and $x = 6$.
In the interval $[0,6]$, $y = 12x - 2x^2$ is above $y = x^2 - 6x$.
$A'(x) = 12x - 2x^2 - x^2 + 6x = 18x - 3x^2$.
$F(x) = 9x^2 - x^3$. $A(6) = F(6) - F(0) = 324 - 216 = 108$.

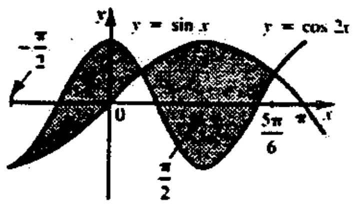


(b) The points of intersection occur when $\frac{4}{x^2} = 5 - x^2$, $x \neq 0$
$\Rightarrow 4 = 5x^2 - x^4 \Rightarrow x^4 - 5x^2 - 4 = 0 \Rightarrow (x^2-4)(x^2-1) = 0$
$\Rightarrow x = \pm 2$, $x = \pm 1$. Two regions of equal area are determined.
The curve $y = 5 - x^2$ is the upper boundary in both cases.
Two times A_1, the region determined by the interval $[1,2]$ gives us the total area. $A'_1(x) = 5 - x^2 - \frac{4}{x^2}$. $F_1(x) = 5x - \frac{1}{3}x^3 + \frac{4}{x}$.



$A_1(2) = F_1(2) - F_1(1) = 10 - \frac{8}{3} + 2 - 5 + \frac{1}{3} - 4 = \frac{2}{3}$. $A = \frac{4}{3}$.

(c) The points of intersection occur when $\sin x = \cos 2x$
$\Rightarrow \sin x = 1 - 2\sin^2 x \Rightarrow 2\sin^2 x + \sin x - 1 = 0$
$\Rightarrow (2\sin x - 1)(\sin x + 1) = 0 \Rightarrow \sin x = \frac{1}{2}$ and $x = \frac{\pi}{6}, \frac{5\pi}{6}$
or $\sin x = -1$ and $x = -\frac{\pi}{2}$. Two regions are determined.
In A_1, $y = \cos 2x$ is above $y = \sin x$, and in A_2,
$y = \sin x$ is above $y = \cos 2x$. $A_1'(x) = \cos 2x - \sin x$.

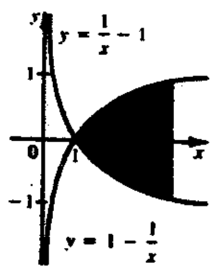


$F_1(x) = \frac{1}{2}\sin 2x + \cos x$. $A_1(\frac{\pi}{6}) = F_1(\frac{\pi}{6}) - F_1(-\frac{\pi}{2}) = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{4}$.

$A_2'(x) = \sin x - \cos 2x$. $F_2(x) = -\cos x - \frac{1}{2}\sin 2x$. $A_2(\frac{5\pi}{6}) = F_2(\frac{5\pi}{6}) - F_2(\frac{\pi}{6})$

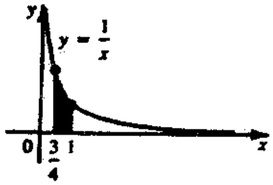
$= \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{4} = \frac{3\sqrt{3}}{2}$. Therefore $A = \frac{3\sqrt{3}}{4} + \frac{3\sqrt{3}}{2} = \frac{9\sqrt{3}}{4}$.

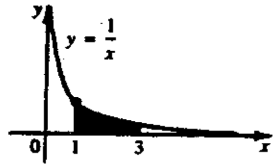
(d) The points of intersection occur when $\frac{1}{x} - 1 = 1 - \frac{1}{x}$
$\Rightarrow \frac{2}{x} = 2 \Rightarrow x = 1$. One region is determined
and $y = 1 - \frac{1}{x}$ is the upper boundary.
$A'(x) = 1 - \frac{1}{x} - \frac{1}{x} + 1 = 2 - \frac{2}{x}$. $F(x) = 2x - 2\ln x$.
$A(4) = F(4) - F(1) = 8 - 2\ln 4 - 2 + 2\ln 1 = 6 - \ln 16$.



Review Exercise 10.6

5. (a) $-\ln\frac{3}{4}$ (b) $\ln 12 - \ln 4 = \ln 3$





6. (a) R is the region under $y = \frac{1}{x}$ from 4 to 12. Therefore $A = \ln 12 - \ln 4 = \ln 3$.

   (b) The area in part (a) is less than the area of the trapezoid with vertices $(4,0)$, $(12,0)$, $(12,\frac{1}{12})$, $(4,\frac{1}{4})$. The area of the trapezoid is $\frac{1}{2}(12-4)(\frac{1}{12}+\frac{1}{4})$ $= \frac{4}{3} = 1.333333$. Therefore $\ln 3 < 1.333333$.

7. $A = \ln 3 = \ln(\frac{3}{2} \times 2) = \ln 2 + \ln\frac{3}{2} = \ln 2 + \ln(\frac{2}{3})^{-1} = \ln 2 - \ln\frac{2}{3} = A_1 + A_2$.

8. (a) The region lies below the x-axis.

$$\Delta x = \frac{2}{n};\ x_i = \frac{2i}{n};\ \sum_{i=1}^{n}\left[\frac{2}{n}f\left(\frac{2i}{n}\right)\right] = \frac{2}{n}\sum_{i=1}^{n}\left[\frac{4i^2}{n^2} - \frac{4i}{n}\right]$$

$$= \frac{8}{n^3} \times \frac{n(n+1)(2n+1)}{6} - \frac{8}{n^2} \times \frac{n(n+1)}{2}$$

The negative of the limit as $n \to \infty$ gives us the area.

$$\text{Area} = -\frac{4}{3}\lim_{n\to\infty}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) + 4\lim_{n\to\infty}\left(1+\frac{1}{n}\right) = -\frac{8}{3} + 4 = \frac{4}{3}$$

(b) $\Delta x = \frac{3-1}{n} = \frac{2}{n};\ x_i = 1 + \frac{2i}{n};$

$$\sum_{i=1}^{n}\left[\frac{2}{n}f\left(1+\frac{2i}{n}\right)\right] = \frac{2}{n}\sum_{i=1}^{n}\left[1 + \frac{6i}{n} + \frac{12i^2}{n^2} + \frac{8i^3}{n^3} - 1\right]$$

$$= \frac{12}{n^2} \times \frac{n(n+1)}{2} + \frac{24}{n^3} \times \frac{n(n+1)(2n+1)}{6} + \frac{16}{n^4} \times \frac{n^2(n+1)^4}{4}$$

$$\text{Area} = 6\lim_{n\to\infty}\left(1+\frac{1}{n}\right) + 4\lim_{n\to\infty}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) + 4\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^2 = 6 + 8 + 4 = 18$$

(c) $\Delta x = \frac{6-2}{n} = \frac{4}{n}$; $x_i = 2 + \frac{4i}{n}$;

$$\sum_{i=1}^{n}\left[\frac{4}{n}f\left(2+\frac{4i}{n}\right)\right] = \frac{4}{n}\sum_{i=1}^{n}\left[\left(2+\frac{4i}{n}\right)^3 + \left(2+\frac{4i}{n}\right)^2 + 1\right]$$

$$= \frac{4}{n}\sum_{i=1}^{n}\left[8 + \frac{48i}{n} + \frac{96i^2}{n^2} + \frac{64i^3}{n^3} + 4 + \frac{16i}{n} + \frac{16i^2}{n^2} + 1\right]$$

$$= \frac{4}{n}\sum_{i=1}^{n}\left[13 + \frac{64i}{n} + \frac{112i^2}{n^2} + \frac{64i^3}{n^3}\right]$$

$$= 52 + \frac{256}{n^2} \times \frac{n(n+1)}{2} + \frac{448}{n^3} \times \frac{n(n+1)(2n+1)}{6} + \frac{256}{n^4} \times \frac{n^2(n+1)^2}{4}$$

Area $= 52 + 128\lim_{n\to\infty}\left(1+\frac{1}{n}\right) + \frac{448}{6}\lim_{n\to\infty}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) + 64\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^2$

$$= 52 + 128 + \frac{448}{3} + 64 = 393\tfrac{1}{3}.$$

9. (a) $A \doteq \frac{1}{2} \times \frac{1}{2}\left[\left(1 + \frac{1}{2.25}\right) + \left(\frac{1}{2.25} + \frac{1}{4}\right) + \left(\frac{1}{4} + \frac{1}{6.25}\right) + \left(\frac{1}{6.25} + \frac{1}{9}\right) + \left(\frac{1}{9} + \frac{1}{12.25}\right)\right]$

$+ \frac{1}{2} \times \frac{1}{2}\left[\left(\frac{1}{12.25} + \frac{1}{16}\right)\right] = \frac{1}{4}\left(1 + \frac{2}{2.25} + \frac{1}{2} + \frac{2}{6.25} + \frac{2}{9} + \frac{2}{12.25} + \frac{1}{16}\right) \doteq 0.789219.$

(b) $A \doteq \frac{1}{2} \times \frac{\pi}{6}\left[\left(\frac{\frac{\pi}{4}}{\sin\frac{\pi}{4}} + \frac{\frac{5\pi}{12}}{\sin\frac{5\pi}{12}}\right) + \left(\frac{\frac{5\pi}{12}}{\sin\frac{5\pi}{12}} + \frac{\frac{7\pi}{12}}{\sin\frac{7\pi}{12}}\right)\right]$

$+ \frac{1}{2} \times \frac{\pi}{6}\left[\left(\frac{\frac{7\pi}{12}}{\sin\frac{7\pi}{12}} + \frac{\frac{3\pi}{4}}{\sin\frac{3\pi}{4}}\right)\right]$

$$= \frac{\pi}{12}\left(\frac{\pi}{4\sin\frac{\pi}{4}} + \frac{10\pi}{12\sin\frac{5\pi}{12}} + \frac{14\pi}{12\sin\frac{7\pi}{12}} + \frac{3\pi}{4\sin\frac{3\pi}{4}}\right) \doteq 2.866105$$

10. The base of each rectangle is 1. $A \doteq \frac{e^{\frac{3}{2}}}{\frac{3}{2}} + \frac{e^{\frac{5}{2}}}{\frac{5}{2}} + \frac{e^{\frac{7}{2}}}{\frac{7}{2}} + \frac{e^{\frac{9}{2}}}{\frac{9}{2}}$

$= 2e^{\frac{3}{2}}\left(\frac{1}{3} + \frac{e}{5} + \frac{e^2}{7} + \frac{e^3}{9}\right) \doteq 37.326155.$

11. $\ln 10$ is the area under $y = \frac{1}{x}$ from 1 to 10. The base of each rectangle is $\frac{9}{4}$. We use the midpoint of the base to determine the height of each rectangle.

$A \doteq \frac{9}{4}\left(\frac{8}{17} + \frac{8}{35} + \frac{8}{53} + \frac{8}{71}\right) \doteq 2.166253$

12. (a) $\Delta x = \frac{1}{n}$; $x_i = \frac{i}{n}$; $\sum_{i=1}^{n}\left[\frac{1}{n}f\left(\frac{i}{n}\right)\right] = \frac{1}{n}\sum_{i=1}^{n}\frac{i^5}{n^5} = \frac{1^5 + 2^5 + 3^5 + \cdots + n^5}{n^6}$.

Now $\lim_{n \to \infty} \frac{1}{n}\sum_{i=1}^{n}\frac{i^5}{n^5}$ is the area under $y = x^5$ from 0 to 1. We calculate this area using an antiderivative. $A'(x) = x^5$. $F(x) = \frac{1}{6}x^6$. $A(1) = F(1) - F(0) = \frac{1}{6}$.

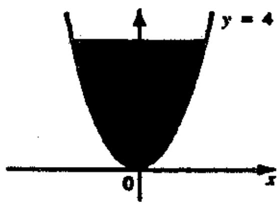
Therefore $\lim_{n \to \infty} \frac{1^5 + 2^5 + 3^5 + \cdots + n^5}{n^6} = \frac{1}{6}$

(b) $\lim_{n \to \infty} \frac{1}{n^4}(1^3 + 2^3 + 3^3 + \cdots + n^3)$ is the area under $y = x^3$ from 0 to 1.

$A'(x) = x^3$. $F(x) = \frac{1}{4}x^4$. $A(1) = F(1) - F(0) = \frac{1}{4}$.

Therefore $\lim_{n \to \infty} \frac{1}{n^4}(1^3 + 2^3 + 3^3 + \cdots + n^3) = \frac{1}{4}$.

13. (a) The points of intersection occur when $x^2 = 4 \Rightarrow x = \pm 2$.
The line $y = 4$ is the upper boundary .
$A'(x) = 4 - x^2$. $F(x) = 4x - \frac{1}{3}x^3$.
$A(2) = F(2) - F(-2)$
$= 8 - \frac{8}{3} + 8 - \frac{8}{3} = \frac{32}{3}$.

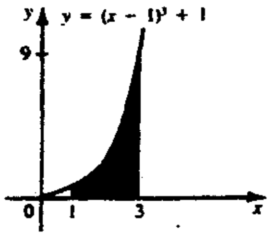


(b) The points of intersection occur when $x^2 = c \Rightarrow x = \pm\sqrt{c}$.
The line $y = c$ is the upper boundary.
$A'(x) = c - x^2$. $F(x) = cx - \frac{1}{3}x^3$.
$A(\sqrt{c}) = F(\sqrt{c}) - F(-\sqrt{c})$

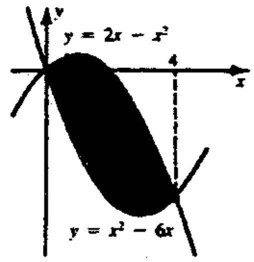
$= c\sqrt{c} - \frac{1}{3}c\sqrt{c} + c\sqrt{c} - \frac{1}{3}c\sqrt{c} = \frac{4}{3}c\sqrt{c}$.

We want $\frac{4}{3}c\sqrt{c} = \frac{16}{3} \Rightarrow c^{\frac{3}{2}} = 4 \Rightarrow c = 4^{\frac{2}{3}} = \sqrt[3]{16} = 2\sqrt[3]{2}$.

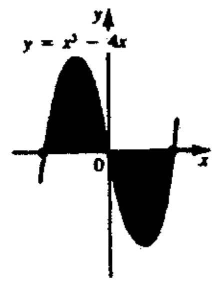
10.7 CHAPTER 10 TEST



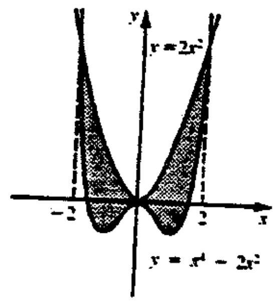
1. $A'(x) = (x-1)^3 + 1$. $F(x) = \frac{1}{4}(x-1)^4 + x$.
   $A(3) = F(3) - F(1) = 4 + 3 - 1 = 6$.



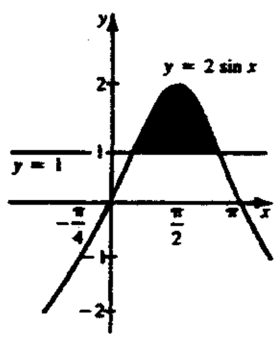
2. The points of intersection occur when $x^2 - 6x = 2x - x^2$
   $\Rightarrow 2x(x-4) = 0 \Rightarrow x = 0,4$. The upper boundary is $y = 2x - x^2$.
   $A'(x) = 2x - x^2 - x^2 + 6x = 8x - 2x^2$.
   $F(x) = 4x^2 - \frac{2}{3}x^3$. $A(4) = F(4) - F(0) = 64 - \frac{128}{3} = \frac{64}{3}$.



3. $x^3 - 4x = 0 \Rightarrow x(x-2)(x+2) = 0 \Rightarrow$ x-intercepts are -2, 0 and 2.
   Two equal regions are determined. We calculate A_1, the area above the x-axis from -2 to 0 and double it.
   $A_1'(x) = x^3 - 4x$. $F_1(x) = \frac{1}{4}x^4 - 2x^2$. $A_1(0) = F_1(0) - F_1(-2) = -4 + 8 = 4$.
   Therefore $A = 2(4) = 8$



4. The points of intersection occur when $x^4 - 2x^2 = 2x^2$
   $\Rightarrow x^2(x^2 - 4) = 0 \Rightarrow x = 0, \pm 2$. Two equal regions are determined.
   We calculate A_1, the area below $y = 2x^2$ and above $y = x^4 - 2x^2$ from 0 to 2 and double it. $A'_1(x) = 2x^2 - x^4 + 2x^2 = 4x^2 - x^4$.
   $F(x) = \frac{4}{3}x^3 - \frac{1}{5}x^5$. $A_1(2) = F_1(2) - F_1(0) = \frac{32}{3} - \frac{32}{5} = \frac{64}{15}$.
   Therefore $A = \frac{128}{15}$.



5. The points of intersection occur when $2\sin x = 1$ in the interval $[-\frac{\pi}{4}, \pi] \Rightarrow \sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$.
   Three regions are determined. We calculate the area of each one separately and add the results.
   A_1 is the region below $y = 1$ and above $y = 2\sin x$ from $-\frac{\pi}{4}$ to $\frac{\pi}{6}$.
   A_2 is below $y = 2\sin x$ and above $y = 1$ from $\frac{\pi}{6}$ to $\frac{5\pi}{6}$.
   A_3 is below $y = 1$ and above $y = 2\sin x$ from $\frac{5\pi}{6}$ to π.

   $A'_1(x) = 1 - 2\sin x$. $F_1(x) = x + 2\cos x$.
   $A_1(\frac{\pi}{6}) = F_1(\frac{\pi}{6}) - F_1(-\frac{\pi}{4}) = \frac{\pi}{6} + \sqrt{3} - \frac{\pi}{4} - \sqrt{2}. = \frac{\pi}{12} + \sqrt{3} - \sqrt{2}$.

   $A'_2(x) = 2\sin x - 1$. $F_2(x) = -2\cos x - x$. $A_2(\frac{5\pi}{6}) = F_2(\frac{5\pi}{6}) - F_2(\frac{\pi}{6})$
   $= \sqrt{3} - \frac{5\pi}{6} + \sqrt{3} + \frac{\pi}{6} = 2\sqrt{3} - \frac{2\pi}{3}$.

5. $A'_3(x) = 1 - 2\sin x$. $F_3(x) = x + 2\cos x$.
$A_3(\pi) = F_3(\pi) - F_3(\frac{5\pi}{6}) = \pi - 2 - \frac{5\pi}{6} + \sqrt{3} = \frac{\pi}{6} + \sqrt{3} - 1$.
Therefore $A = 4\sqrt{3} - \sqrt{2} - 1 - \frac{5\pi}{12}$.

6. (a) R is the region under $y = \frac{1}{x}$ from 2 to 5. $A = \ln 5 - \ln 2 = \ln 2.5$.

(b) The area in part (a) is less than the area of the trapezoid with vertices $(2,0)$, $(5,0)$, $(5,\frac{1}{5})$ and $(2,\frac{1}{2})$. the area of the trapezoid is $\frac{1}{2}(5-2)(\frac{1}{2}+\frac{1}{5})$ $= \frac{3}{2} \times \frac{7}{10} = \frac{21}{20} = 1.05$. Therefore $\ln 2.5 < 1.05$.

7. $\Delta x = \frac{3}{n}$; $x_i = \frac{3i}{n}$; $\sum_{i=1}^{n}\left[\frac{3}{n}f\left(\frac{3i}{n}\right)\right] = \frac{3}{n}\sum_{i=1}^{n}\left[\left(\frac{3i}{n}\right)^2 + 2\left(\frac{3i}{n}\right)\right]$

$$= \frac{27}{n^3} \times \frac{n(n+1)(2n+1)}{6} + \frac{18}{n^2} \times \frac{n(n+1)}{2}.$$

$$A = \frac{9}{2}\lim_{n\to\infty}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) + 9\lim_{n\to\infty}\left(1+\frac{1}{n}\right) = 9 + 9 = 18.$$

8. $$A \doteq \frac{1}{2} \times \frac{\pi}{8}\left[\left(0 + \frac{\pi}{8}\cos\frac{\pi}{8}\right) + \left(\frac{\pi}{8}\cos\frac{\pi}{8} + \frac{\pi}{4}\cos\frac{\pi}{4}\right) + \left(\frac{\pi}{4}\cos\frac{\pi}{4} + \frac{3\pi}{8}\cos\frac{3\pi}{8}\right)\right]$$
$$+ \frac{1}{2} \times \frac{\pi}{8}\left(\frac{3\pi}{8}\cos\frac{3\pi}{8} + \frac{\pi}{2}\cos\frac{\pi}{2}\right)$$

$$= \frac{\pi}{16}\left(\frac{\pi}{4}\cos\frac{\pi}{8} + \frac{\pi}{2}\cos\frac{\pi}{4} + \frac{3\pi}{4}\cos\frac{3\pi}{8}\right) \doteq 0.537607.$$

9. The base of each rectangle is $\frac{4}{3}$. The midpoints of the bases are $\frac{8}{3}$, $\frac{12}{3}$, and $\frac{16}{3}$.

$$A \doteq \frac{4}{3}\left[\frac{e^{\frac{8}{3}}}{\frac{8}{3}} + \frac{e^{\frac{12}{3}}}{\frac{12}{3}} + \frac{e^{\frac{16}{3}}}{\frac{16}{3}}\right]$$

$$= 4e^{\frac{8}{3}}\left[\frac{1}{8} + \frac{1}{12}e^{\frac{4}{3}} + \frac{1}{16}e^{\frac{8}{3}}\right] \doteq 77.177154.$$

10. The functions do not intersect.

Let $f(x) = |x^2 - 1|$ and $g(x) = -|x|$.

$$f(x) = \begin{cases} x^2 - 1, & x < -1 \\ 1 - x^2, & -1 \leq x \leq 1 \\ x^2 - 1, & x > 1 \end{cases} \qquad g(x) = \begin{cases} x, & x < 0 \\ -x, & x \geq 0 \end{cases}$$

Four regions A_1, A_2, A_3, and A_4 must be calculated and the results added.

$A'_1(x) = x^2 - 1 - x$. $F_1(x) = \frac{1}{3}x^3 - x - \frac{1}{2}x^2$.

$A_1(-1) = F_1(-1) - F_1(-2)$

$= -\frac{1}{3} + 1 - \frac{1}{2} + \frac{8}{3} - 2 + 2 = \frac{17}{6}$

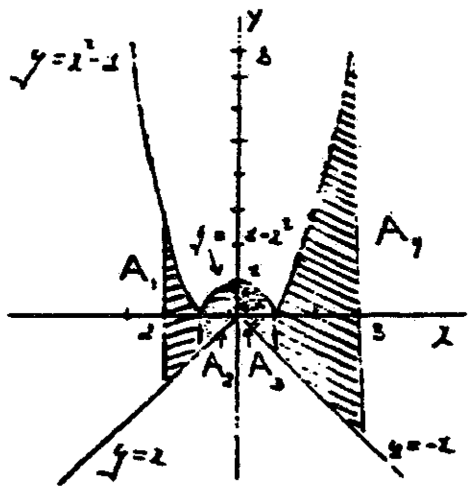
$A'_3(x) = 1 - x^2 + x$. $F_3(x) = x - \frac{1}{3}x^3 + \frac{1}{2}x^2$.

$A_3(1) = F_3(1) - F_3(0) = 1 - \frac{1}{3} + \frac{1}{2} = \frac{7}{6} = A_2$

$A'_4(x) = x^2 - 1 + x$. $F_4(x) = \frac{1}{3}x^3 - x + \frac{1}{2}x^2$.

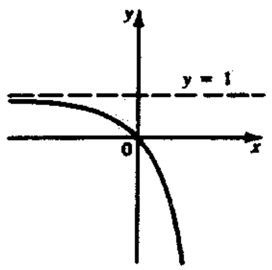
$A_4(3) = F_4(3) - F_4(1)$

$= 9 - 3 + \frac{9}{2} - \frac{1}{3} + 1 - \frac{1}{2} = \frac{32}{3}$. Therefore $A = \frac{17}{6} + 2(\frac{7}{6}) + \frac{32}{3} = \frac{95}{6}$.



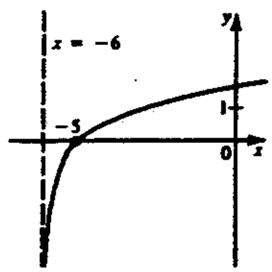
CUMULATIVE REVIEW FOR CHAPTERS 8 TO 10

1. (a)



(b) Domain: R
Range: $(-\infty, 1)$
Asymptote: $y = 1$

2. (a)



(b) Domain: $(-6, \infty)$
Range: R
Asymptote: $x = -6$

3. (a) $\lim_{x \to \infty}(1 + e^{-x^2}) = 1 + 0 = 1$, since $-x^2 \to -\infty$ as $x \to \infty$.

(b) $\lim_{x \to 0^+} \ln(\sin x) = -\infty$, since $\sin x \to 0^+$ as $x \to 0^+$.

4. (a) $\ln(e^2) = 2$ (b) $e^{2\ln 3} = e^{\ln 9} = 9$

5. (a) $e^{2x+1} = 20 \Rightarrow 2x + 1 = \ln 20 \Rightarrow x = \frac{1}{2}(\ln 20 - 1) \doteq 0.997860$

(b) $\ln(1 - x) = -2 \Rightarrow 1 - x = e^{-2} \Rightarrow x = 1 - e^{-2} \doteq 0.864665$

6. (a) $y = (x+1)e^{3-4x}$, $y' = e^{3-4x} + (x+1)e^{3-4x}(-4) = -(3+4x)e^{3-4x}$

(b) $y = \frac{\ln(x^2+1)}{x}$, $y' = \frac{x\left(\frac{2x}{x^2+1}\right) - \ln(x^2+1)}{x^2} = \frac{2x^2 - (x^2+1)\ln(x^2+1)}{x^2(x^2+1)}$

(c) $y = \ln(1 + e^{x^2})$, $y' = \frac{2xe^{x^2}}{1 + e^{x^2}}$

(d) $y = 10^{-\sqrt{x}}$, $y' = 10^{-\sqrt{x}}\left(-\frac{1}{2\sqrt{x}}\right)\ln 10 = -\frac{10^{-\sqrt{x}}(\ln 10)}{2\sqrt{x}}$

(e) $y = \ln\sqrt{\frac{x}{1-x^3}} = \frac{1}{2}[\ln x - \ln(1-x^3)]$, $y' = \frac{1}{2}\left(\frac{1}{x} - \frac{-3x^2}{1-x^3}\right) = \frac{1}{2}\left(\frac{1}{x} + \frac{3x^2}{1-x^3}\right)$

(f) $y = x^{\tan x}$, $\ln y = \tan x \ln x \Rightarrow \frac{y'}{y} = \sec^2 x \ln x + \frac{\tan x}{x} \Rightarrow$

$y' = x^{\tan x}(\sec^2 x \ln x + \frac{\tan x}{x})$

7. (a) $g(x) = e^{f(x)} \Rightarrow g'(x) = e^{f(x)}f'(x)$ (b) $h(x) = f(e^x) \Rightarrow h'(x) = f'(e^x)e^x$

8. $y = e^x$, $y' = e^x$, so the tangent at (a, e^a) has slope e^a. If this tangent is parallel to $3x - y = 6$, then $e^a = 3 \Rightarrow a = \ln 3$. Thus the point of contact is $(\ln 3, 3)$ and the equation of the tangent is $y - 3 = 3(x - \ln 3)$ or $3x - y = 3\ln 3 - 3$.

9. (a) $f(x) = xe^x$, $f'(x) = e^x + xe^x = (x+1)e^x > 0$ when $x > -1$. So f increases on $(-1, \infty)$ and decreases on $(-\infty, -1)$.
(b) The local and absolute minimum is $f(-1) = -e^{-1} = -\frac{1}{e}$.
(c) $f''(x) = e^x + (1+x)e^x = (x+2)e^x > 0$ when $x > -2$. So f is CU on $(-2, \infty)$ and CD on $(-\infty, -2)$. The inflection point is $(-2, -2e^{-2})$.

10. Let F denote the general antiderivative of f.
(a) $F(x) = 12(\frac{1}{4}x^4) - 9(\frac{1}{3}x^3) + 8(\frac{1}{2}x^2) + 31x + C = 3x^4 - 3x^3 + 4x^2 + 31x + C$
(b) $F(x) = 4(-\frac{1}{2}\cos 2x) + 5(\frac{1}{3}\sin(3x+1)) + C = -2\cos 2x + \frac{5}{3}\sin(3x+1) + C$
(c) $F(x) = -2(\frac{1}{3}e^{3x}) + \frac{1}{3}\left[\frac{1}{-4}e^{-4x}\right] + C = -\frac{2}{3}e^{3x} - \frac{1}{12}e^{-4x} + C$

11. (a) $F(x) = \sqrt{2}\ln(x+1) - \sqrt{3}\ln x + C$
(b) Since $f(x) = (\sqrt{2} + \sqrt{5} + \sqrt{8})x^{\frac{1}{2}}$, we have $F(x) = (\sqrt{2} + \sqrt{5} + \sqrt{8})\frac{1}{\frac{3}{2}}x^{\frac{3}{2}} + C$
So $F(x) = \frac{2}{3}(\sqrt{2} + \sqrt{5} + \sqrt{8})x\sqrt{x} + C$

12. We find the general antiderivative F, then use $F(2) = 3$ to determine the constant.
(a) $F(x) = x^3 + x^2 + C$. So $3 = 2^3 + 2^2 + C$. Thus $C = -9$ and $F(x) = x^3 + x^2 - 9$.
(b) $F'(x) = x^{-\frac{1}{2}} - x \Rightarrow F(x) = \frac{1}{\frac{1}{2}}x^{\frac{1}{2}} - \frac{1}{2}x^2 + C = 2x^{\frac{1}{2}} - \frac{1}{2}x^2 + C$. So
$3 = 2\sqrt{2} - \frac{1}{2}2^2 + C$. Hence $C = 5 - 2\sqrt{2}$, so $F(x) = 2\sqrt{x} - \frac{1}{2}x^2 + 5 - 2\sqrt{2}$.
(c) $F(x) = \frac{3}{4}e^{4x} + C$. So $3 = \frac{3}{4}e^8 + C \Rightarrow C = 3 - \frac{3}{4}e^8$ and $F(x) = \frac{3}{4}(e^{4x} - e^8) + 3$.

13. From Question 7 in Exercise 9.3, we know that $t = \frac{v_0 + \sqrt{v_0^2 + 19.6h_0}}{9.8}$. In the present case $v_0 = -20$, $h_0 = 155$, since height is measured upward and the ston...

is hurled down. Thus $t = \frac{-20+\sqrt{3438}}{9.8} = 3.9$ s.

14. (a) Let f(t) = number of bacteria after t hours. Then $f(0) = 1200$ and $f(1) = 4000$. $f(t) = 1200e^{kt} \Rightarrow 1200e^{k} = 4000 \Rightarrow e^{k} = \frac{10}{3} \Rightarrow k = \ln(\frac{10}{3})$. So $f(t) = 1200e^{\ln(\frac{10}{3})t} = 1200(\frac{10}{3})^{t}$.

(b) After 3 h the number is $f(3) = 1200(\frac{10}{3})^3 \doteq 44444$.

(c) After 3 h the rate is $f'(3) = kf(3) = \ln(\frac{10}{3})(44444) \doteq 53510$ bacteria/h.

(d) $f(t) = 1200e^{\ln(\frac{10}{3})t} = 10000 \Rightarrow e^{\ln(\frac{10}{3})t} = \frac{25}{3} \Rightarrow \ln(\frac{10}{3})t = \ln(\frac{25}{3}) \Rightarrow$

$t = \frac{\ln(\frac{25}{3})}{\ln(\frac{10}{3})} \doteq 1.76$ h.

15. (a) Let f(t) = mass after t minutes. Then $f(0) = 50$, so $f(t) = 50e^{kt}$. Then $f(19.7) = 25 \Rightarrow 50e^{19.7k} = 25$, so $e^{19.7k} = \frac{1}{2}$, $19.7k = \ln(\frac{1}{2}) = -\ln 2$, $k = -\frac{\ln 2}{19.7}$ $\Rightarrow f(t) = 50e^{-\frac{\ln 2}{19.7}t}$

(b) After 2 h, the mass is $f(120) = 50e^{-\frac{\ln 2}{19.7}(120)} \doteq 0.73$ g.

(c) The rate is $f'(120) = kf(120) = -\frac{\ln 2}{19.7}f(120) \doteq -0.026$ g/min.

(d) $f(t) = 50e^{-\frac{\ln 2}{19.7}t} = 1 \Rightarrow e^{-\frac{\ln 2}{19.7}t} = \frac{1}{50} \Rightarrow -\frac{\ln 2}{19.7}t = -\ln 50 \Rightarrow$

$t = (19.7)\frac{\ln 50}{\ln 2} \doteq 111$ min = 1h, 51 min.

16. If T, t are the usual variables, then $T = A + (T_0 - A)e^{kt}$ from Newton's Law of Cooling (p. 418). We are given that $T_0 = 150$ (the initial temperature of the bread) and $A = 30$ (the termperature of the cooling rack). So $T = 30 + 120e^{kt}$. Next we are told that $T = 100$ $(= 150 - 50)$ when $t = 3$. So $100 = 30 + 120e^{3k}$. This enables us to find k: $e^{3k} = \frac{70}{120} \Rightarrow k = \frac{1}{3}\ln(\frac{7}{12})$. Finally we find t so that $T = 40$: $40 = 30 + 120e^{kt}$, so $kt = \ln(\frac{10}{120})$. Thus $t = \frac{\ln(\frac{1}{12})}{\frac{1}{3}\ln(\frac{7}{12})} \doteq 14$. In about $14 - 3 = 11$ min, the bread reaches 40°C.

17. From Question 8 in Exercise 9.5, we have $A = cV + (A_0 - cV)e^{-\frac{rt}{V}}$. Here $A_0 = 3$, $V = 450$, $c = 17 \times 10^{-3}$, $r = 6$ (in the appropriate units). Thus

$A = 7.65 - 4.65e^{-\frac{t}{75}}$. When $t = 30$, $A \doteq 4.53$, so there is about 4.53 kg of salt in the tank after 30 min.

18. We use the logistic equation since this is a situation involving inhibited growth. Let P denote the number of fish at time t (in years). Since $P_0 = 200$ and $K = 6000$, we have $P = \frac{6000}{1+29e^{-kt}}$. But P = 600 when t = 1, so $600 = \frac{6000}{1+29e^{-k}}$. This gives $e^{-k} = \frac{9}{29}$, $k = \ln(\frac{29}{9})$. We want t so that $3000 = \frac{6000}{1+29e^{-kt}}$. Thus $e^{-kt} = \frac{1}{29} \Rightarrow -kt = -\ln 29 \Rightarrow t = \frac{\ln 29}{\ln(\frac{29}{9})} \doteq 2.9$. This tells us that in about two years and 11 months after the initial seeding of the lake, there will be 3000 fish.

19. Since $1.2^2 = 1.44$, we see that $s = A\cos 1.2t + B\sin 1.2t$, where A, B are constants to be determined from the initial conditions. At t = 0, s = 1.7, so A = 1.7. Next we have $\frac{ds}{dt} = -1.2A\sin 1.2t + 1.2B\cos 1.2y$. At t = 0, $\frac{ds}{dt} = 1.8$, so 1.8 = 1.2B. Hence B = 1.5 and $s = 1.7\cos 1.2t + 1.5\sin 1.2t$.

20. In general $\frac{d^2s}{dt^2} + \frac{k}{m}s = 0$ where s is the displacement. (Recall that our convention is to use "up" as positive.) We find the spring constant using $k(0.80 - 0.50) = 9$. So k = 30. Since m = 1.2, the differential equation for the displacement is $\frac{d^2s}{dt^2} + 25s = 0$. This gives $s = A\cos 5t + B\sin 5t$. Initially, $s_0 = 0.5 - 0.35 = 0.15$ and $v_0 = 0$. So $s = 0.15\cos 5t$.

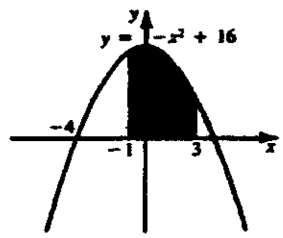
(a) $s = 0.15 \Leftrightarrow \cos 5t = 1 \Leftrightarrow 5t = 2n\pi$. So the time t of first return to the initial position is $5t = 2\pi$, $t = \frac{2\pi}{5}$ s.

(b) Since $v = \frac{ds}{dt} = -0.75\sin 5t$, the speed is $|v| = 0.75|\sin 5t|$. The displacement is 0 when $\cos 5t = 0$. In that case $|\sin 5t| = 1$ since $\cos^2 x + \sin^2 x = 1$. Hence $|v| = 0.75$ m/s when the displacement is 0.

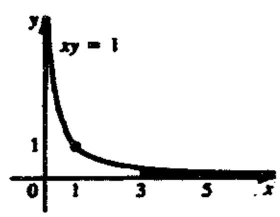
21. (a) $A'(x) = 16 - x^2$. $F(x) = 16x - \frac{1}{3}x^3$.

$A(3) = F(3) - F(-1) = 48 - 9 + 16 - \frac{1}{3}$

$= \frac{164}{3}$



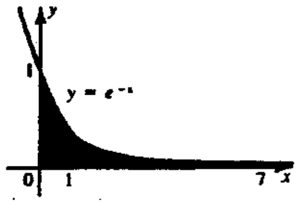
(b) $A = \ln 5 - \ln 3 = \ln\frac{5}{3}$



(c) $A'(x)=e^{-x}$. $F(x)=-e^{-x}$.

$A(7)=F(7)-F(0)=-e^{-7}+1$

$=\frac{e^7-1}{e^7}$.



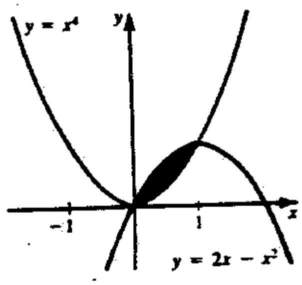
22. (a) Points of intersection occur when $x^4=2x-x^2$

$\Rightarrow x^4+x^2-2x=0 \Rightarrow x(x-1)(x^2+x+2)$

$=0 \Rightarrow x=0, 1.$

$A'(x)=2x-x^2-x^4$. $F(x)=x^2-\frac{1}{3}x^3-\frac{1}{5}x^5$

$A(1)=F(1)-F(0)=1-\frac{1}{3}-\frac{1}{5}=\frac{7}{15}$.



(b) Points of intersection occur when

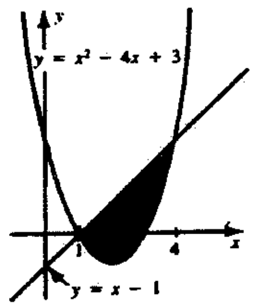
$x^2-4x+3=x-1$

$\Rightarrow x^2-5x-4=0 \Rightarrow (x-1)(x-4)=0 \Rightarrow x=1, 4.$

$A'(x)=x-1-(x^2-4x+3)=-x^2+5x-4$.

$F(x)=-\frac{1}{3}x^3+\frac{5}{2}x^2-4x$.

$A(4)=F(4)-F(1)=-\frac{64}{3}+40-16+\frac{1}{3}-\frac{5}{2}+4=\frac{9}{2}$



(c) Points of intersection occur when

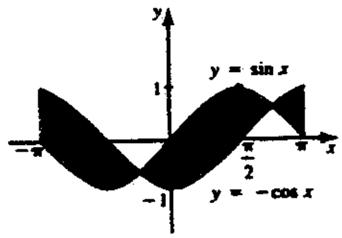
$\sin x=-\cos x \Rightarrow \tan x=-1 \Rightarrow x=-\frac{\pi}{4}, \frac{3}{4}\pi.$

$A=A_1+A_2+A_3=2A_2$.

$A'_2(x)=\sin x+\cos x$. $F_2(x)=-\cos x+\sin x$.

$A_2(\frac{3}{4}\pi)=F_2(\frac{3}{4}\pi)-F_2(-\frac{\pi}{4})$

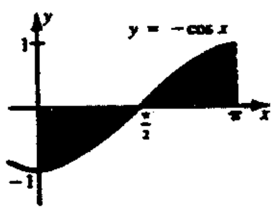
$=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}=\frac{4}{\sqrt{2}}$. Therefore $A=\frac{8}{\sqrt{2}}$.



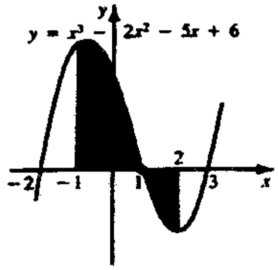
(d) The x-intercept is $\frac{\pi}{2}$. $A=A_1+A_2=2A_2$.

$A'_2(x)=-\cos x$. $F_2(x)=\sin x$.

$A_2(\frac{\pi}{2})=F_2(\frac{\pi}{2})-F_2(0)=1$. Therefore $A=2$



(e) Two distinct regions are determined. We must calculate each one separately and sum the results.



$A'_1(x) = x^3 - 2x^2 - 5x + 6.$

$F_1(x) = \frac{1}{4}x^4 - \frac{2}{3}x^3 - \frac{5}{2}x^2 + 6x.$

$A_1(1) = F_1(1) - F_1(-1) = \frac{1}{4} - \frac{2}{3} - \frac{5}{2} + 6 - \frac{1}{4} - \frac{2}{3} + \frac{5}{2} + 6 = \frac{32}{3}$

$A'_2(x) = -x^3 + 2x^2 + 5x - 6.$

$F_2(x) = -\frac{1}{4}x^4 + \frac{2}{3}x^3 + \frac{5}{2}x^2 - 6x.$

$A_2(2) = F_2(2) - F_2(1) = -4 + \frac{16}{3} + 10 - 12 + \frac{1}{4} - \frac{2}{3} - \frac{5}{2} + 6 = \frac{29}{12}$

$A = \frac{128}{12} + \frac{29}{12} = \frac{157}{12}.$

23. (a) $\Delta x = \frac{a}{n};\ x_i = \frac{ai}{n};\ \sum_{i=1}^{n}\left[\frac{a}{n}f\left(\frac{ai}{n}\right)\right] = \frac{a}{n}\sum_{i=1}^{n}\left[\frac{mai}{n}\right] = \frac{ma^2}{n^2}\sum_{i=1}^{n} i = \frac{ma^2}{n^2}\left[\frac{n(n+1)}{2}\right]$

$A = \lim_{n\to\infty}\left[\frac{ma^2}{2}\left(1 + \frac{1}{n}\right)\right] = \frac{1}{2}ma^2$

(b) $\Delta x = \frac{2}{n};\ x_i = \frac{2i}{n};\ \sum_{i=1}^{n}\left[\frac{2}{n}f\left(\frac{2i}{n}\right)\right] = \frac{2}{n}\sum_{i=1}^{n}\left[-\frac{4i^2}{n^2} + \frac{4i}{n}\right] = \frac{8}{n^2}\sum_{i=1}^{n}\left[i - \frac{i^2}{n}\right]$

$= \frac{8}{n^2}\left[\frac{n(n+1)}{2} - \frac{n(n+1)(2n+1)}{6n}\right].$ $A = \lim_{n\to\infty}\left[4\left(1 + \frac{1}{n}\right) - \frac{4}{3}\left(1 + \frac{1}{n}\right)\left(2 + \frac{1}{n}\right)\right]$

$= 4 - \frac{8}{3} = \frac{4}{3}$

24. $A \doteq \frac{1}{2}\left[\frac{1}{2}\left(\frac{5}{2}e^{\frac{5}{2}} + 2e^2\right) + \frac{1}{2}\left(\frac{5}{2}e^{\frac{5}{2}} + 3e^3\right) + \frac{1}{2}\left(3e^3 + \frac{7}{2}e^{\frac{7}{2}}\right) + \frac{1}{2}\left(\frac{7}{2}e^{\frac{7}{2}} + 4e^4\right)\right]$

$= \frac{1}{4}\left[2e^2 + 5e^{2.5} + 6e^3 + 7e^{3.5} + 4e^4\right] \doteq 161.601142$

25. $A \doteq \frac{\pi}{4}\left[\frac{\sin\frac{\pi}{8}}{\frac{\pi}{8}} + \frac{\sin\frac{3\pi}{8}}{\frac{3\pi}{8}} + \frac{\sin\frac{5\pi}{8}}{\frac{5\pi}{8}} + \frac{\sin\frac{7\pi}{8}}{\frac{7\pi}{8}}\right]$

$= 2\left[\sin\frac{\pi}{8} + \frac{1}{3}\sin\frac{3\pi}{8} + \frac{1}{5}\sin\frac{5\pi}{8} + \frac{1}{7}\sin\frac{7\pi}{8}\right] \doteq 1.860176$