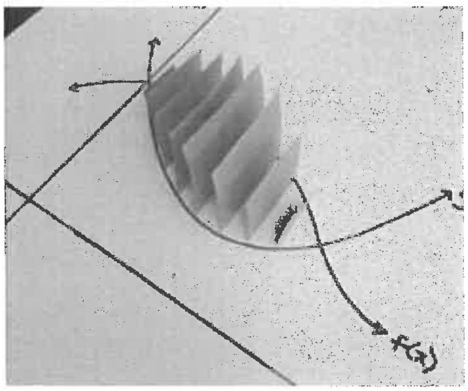
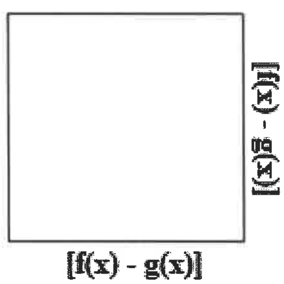
AP Calculus 30L - Mr. Maxwell - Name: ____________

Volumes of Solids with Known Cross Sections

Calculus cannot only be used to find the volume of solids created by revolving two-dimensional shapes around an axis but also the volume of solids formed by cross sections that are geometric shapes. In this lesson, you will derive the formulas for finding volumes of solids given that their cross sections are squares, isosceles right triangles, equilateral triangles, and semi circles.

Cross Sections that are Squares

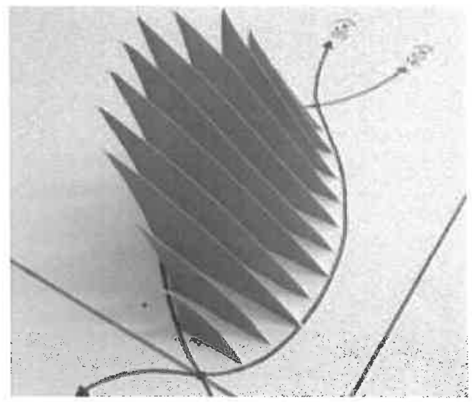


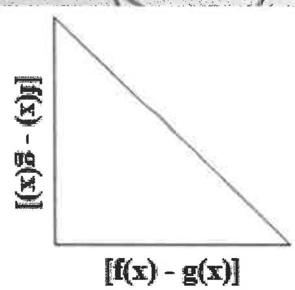


Find the area of the square above in terms of $f(x)$ and $g(x)$.

If all of the cross sectional areas were added up, the total would be the volume of the solid pictured. How do we write this in calculus using an integral?

Cross Sections that are Isosceles Right Triangles

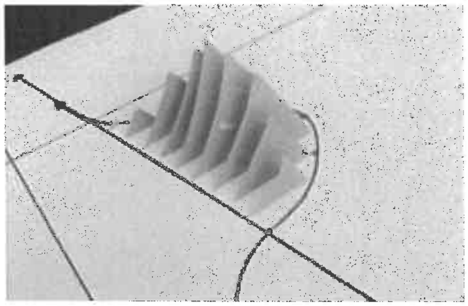


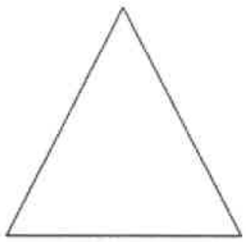


Find the area of the triangle above in terms of $f(x)$ and $g(x)$.

If all of the cross sectional areas were added up, the total would be the volume of the solid pictured. How do we write this in calculus using an integral?

Cross Sections that are Equilateral Triangles



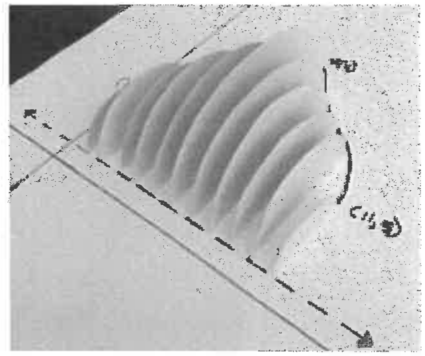


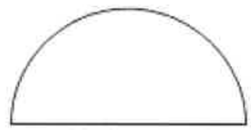
[f(x) - g(x)]

Find the area of the triangle above in terms of $f(x)$ and $g(x)$.

If all of the cross sectional areas were added up, the total would be the volume of the solid pictured. How do we write this in calculus using an integral?

Cross Sections that are Semicircles





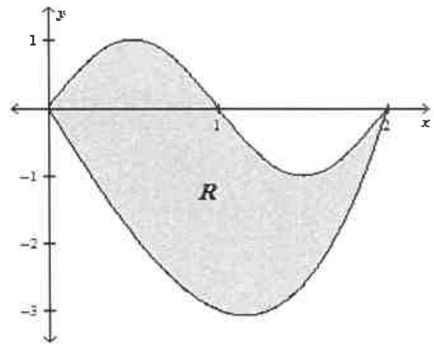
[f(x) - g(x)]

Find the area of the semicircle above in terms of $f(x)$ and $g(x)$.

If all of the cross sectional areas were added up, the total would be the volume of the solid pictured. How do we write this in calculus using an integral?

What do you notice about the integral-defined formulas for finding the volume of solids with certain cross sections?

Region R is bounded by $y = \sin(\pi x)$ and $y = x^3 - 4x$.



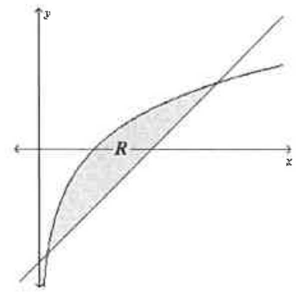
Find the volume of the solids formed whose cross sections are the shapes indicated below. The cross sections are perpendicular to the x – axis.

a. Cross sections are equilateral triangles	b. Cross sections are semi-circles
c. Cross sections are isosceles right triangles	d. Cross sections are squares.
e. Cross sections are rectangles whose height is twice the length of the base.	f. Cross sections are rectangles whose height is one-third the length of the base.

Name____Date______________________Class__________

Day #63 Homework

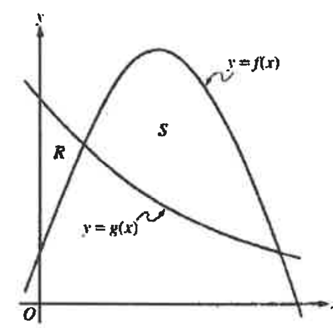
Let R be the region bounded by the graphs of $y = \ln x$ and the line $y = x - 2$ as shown below. Though you may use a calculator, show the integral that you found to arrive at your answer.



1. Find the volume of the solid whose base is region R that is formed by cross sections that are semi-circles that are perpendicular to the x – axis.

2. Find the volume of the solid whose base is region R that is formed by cross sections that are squares that are perpendicular to the x – axis.

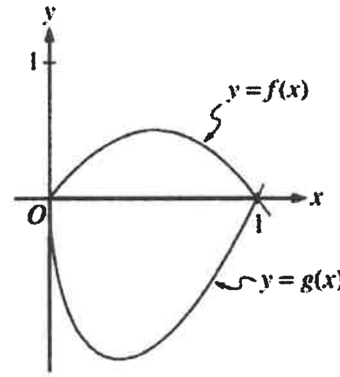
Let f and g be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let R be the region in the first quadrant enclosed by the y-axis and the graphs of f and g, and let S be the region in the first quadrant enclosed by the graphs of f and g shown to the right. Though you may use a calculator, show the integral that you found to arrive at your answer.



3. Find the volume of the solid whose base is the cross section area of region S and is formed by squares that are perpendicular to the x-axis.

4. Find the volume of the solid whose base is the cross section area of region S and is formed by equilateral triangles that are perpendicular to the x – axis.

Let f and g be the functions given by $f(x)=2x(1-x)$ and $g(x)=3(x-1)\sqrt{x}$ for $0 \le x \le 1$. The graphs of f and g are shown in the figure to the right. Though you may use a calculator, show the integral that you found to arrive at your answer.



5. Find the volume of the solid whose base is the cross section of the region bounded by the graphs of f and g and is formed by squares that are perpendicular to the x-axis.

6. Find the volume of the solid whose base is the cross section of the region bounded by the graphs of f and g and is formed by semi-circles that are perpendicular to the x-axis.

7. Find the volume of the solid whose base is the cross section of the region bounded by the graphs of f and g and is formed by equilateral triangles that are perpendicular to the x-axis.

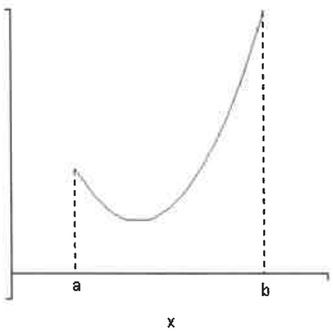
8. Find the volume of the solid whose base is the cross section of the region bounded by the graphs of f and g and is formed by right isosceles triangles that are perpendicular to the x-axis.

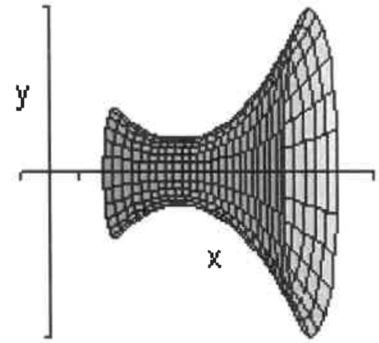
AP Calculus 30L - Mr. Maxwell -

Name: ____________

Volumes of Solids of Revolution

A solid of revolution is formed when a flat, two-dimensional shape is rotated around an axis. Consider the flat region below to the left. When that region is rotated about the $x-$axis, the solid pictured below to the right is formed. This objective of this lesson is to learn to find the volume of such a solid.





Now, imagine slicing the solid into individual discs of height 1 unit. The volume of one of those discs is $V = \pi r^2 h$, or $V = \pi r^2$.

$$\text{Sum of all the discs} = \int_a^b \pi\left[f(x) - \textit{Axis of Rotation}\right]^2 dx$$

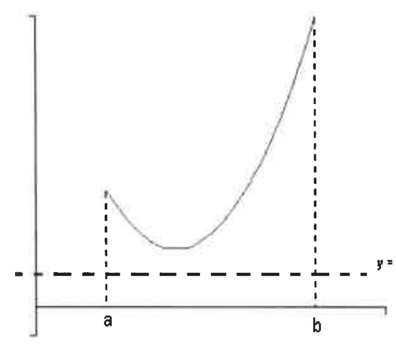
$$\text{Volume of the Solid} = \pi\int_a^b \left[f(x) - \textit{Axis of Rotation}\right]^2 dx$$

Notice that the axis of rotation is the $x-$axis and the bottom function of the region is also the $x-$axis. Imagine for a moment what the solid would look like if the axis of rotation were still the $x-$axis but the bottom function of the region was the line $y = c$. The solid would look similar except for the fact that there would be a cylinder that is cut out of the center.

To find the volume of this solid, we would find the volume of the whole solid that we found previously and then subtract out the solid in the form of a cylinder.

In order to do this, we use the formula below to find the volume of such a solid.

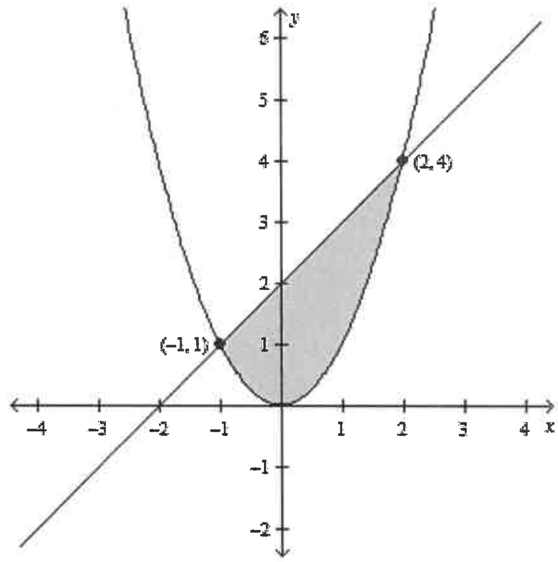
$$\text{Volume} = \pi\int_a^b \left[\textit{OuterFunction} - \textit{axis}\right]^2 - \left[\textit{InnerFunction} - \textit{axis}\right]^2 dx$$



The "outer function" is defined to be the function that is farther from the axis of rotation. The "inner function" is defined to be the function that is closer to the axis of rotation.

If the axis of rotation is the x-axis or is parallel to the x-axis, the integrand needs to be in terms of x and the limits of integration need to be the x-values of the points of intersection of the curves that form the region being rotated.

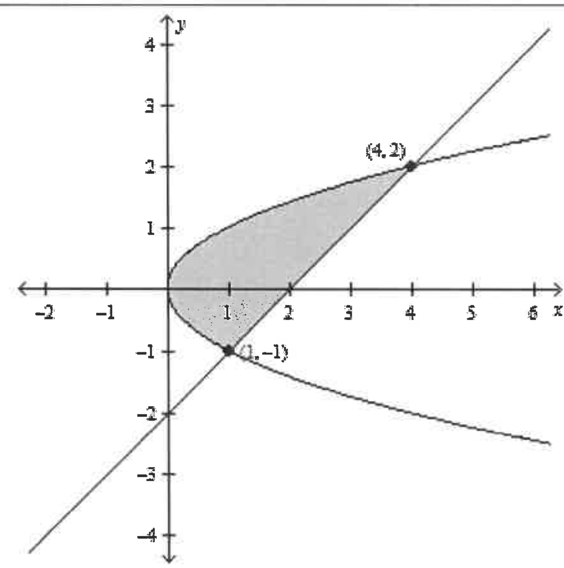
Consider the region pictured to the right that is bounded by the graphs of $y = x^2$ and $y = x + 2$.

Find the volume of the solid formed when the region is rotated about the x – axis.	
Find the volume when the region is rotated about the line $y = 4$.	Find the volume when the region is rotated about the line $y = -2$.

If the axis of rotation is the y-axis or is parallel to the y-axis, the integrand needs to be in terms of y and the limits of integration need to be the y-values of the points of intersection of the curves that form the region being rotated.

Consider the region pictured to the right that is bounded by the graphs $y = \pm\sqrt{x}$ and $y = x - 2$.

Find the volume when the region is rotated about the $y-$axis.



Find the volume when the region is rotated about the line $x = 4$.

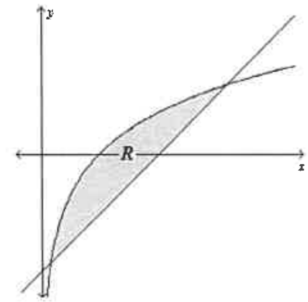
Find the volume when the region is rotated about the line $x = 7$.

Name______________________________________Date__________________Class__________

Day #62 Homework

Let R be the region bounded by the graphs of $y = \ln x$ and the line $y = x - 2$ as shown below. Though you may use a calculator, show the integral that you found to arrive at your answer.

1. Find the coordinates of the points at which the two graphs intersect each other. Then, find the area of R.

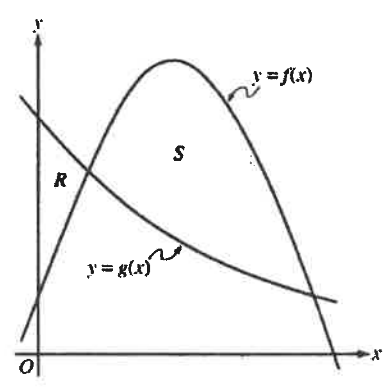


2. Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.

3. Write and evaluate an integral expression that can be used to find the volume of the solid generated when R is rotated about the y-axis.

Let f and g be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let R be the region in the first quadrant enclosed by the y-axis and the graphs of f and g, and let S be the region in the first quadrant enclosed by the graphs of f and g shown to the right. Though you may use a calculator, show the integral that you found to arrive at your answer.

4. Find the points of intersection of f and g.

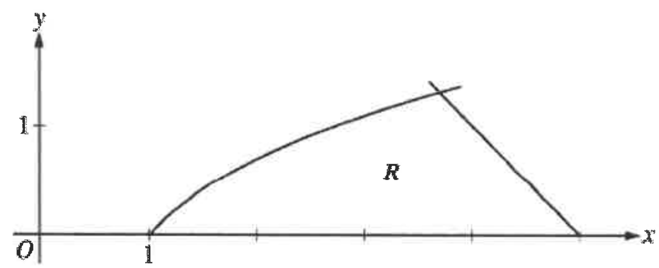


5. Find the area of the region bounded by the graphs of f and g and the x – axis.

4. Find the volume of the solid generated when R is revolved about the horizontal line $y = 8$.

5. Find the volume of the solid generated when S is revolved about the horizontal line $y = -1$.

2012 AP® CALCULUS AB
Question 2



Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.

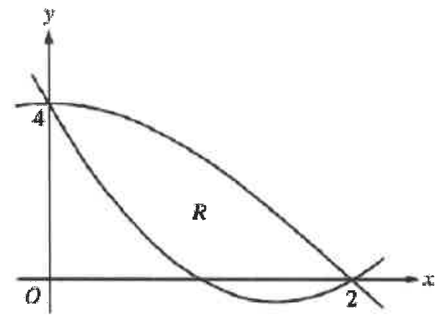
(a) Find the area of R.

(b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

(c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

CALCULATOR NOT PERMITTED

2013 AP® CALCULUS AB
Question 5



Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g, as shown in the figure above.

(a) Find the area of R.

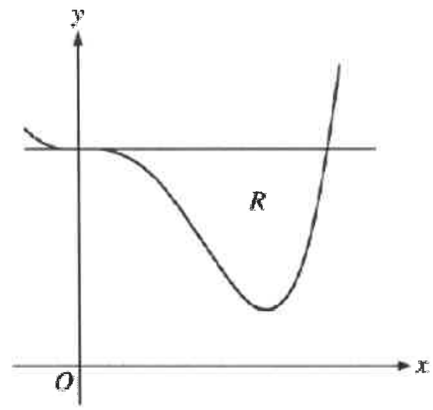
(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

CALCULATOR PERMITTED

2014 AP® CALCULUS AB
Question 2

Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.



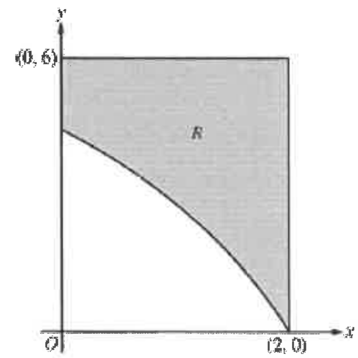
(a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

(b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in R. Find the volume of the solid.

(c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k.

CALCULATOR PERMITTED

2010 AP® CALCULUS AB (Form B)
Question 1



In the figure above, R is the shaded region in the first quadrant bounded by the graph of $y = 4\ln(3 - x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.

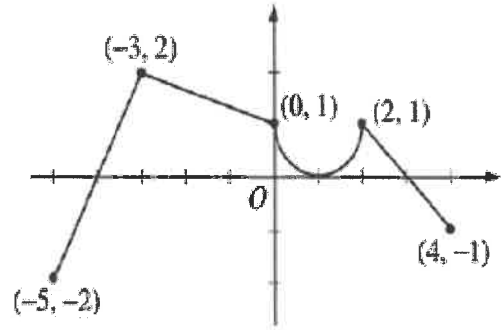
(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 8$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of the solid.

2004 AP® CALCULUS AB
Question 5

The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)\,dt$.



Graph of f

(a) Find $g(0)$ and $g'(0)$.

(b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.

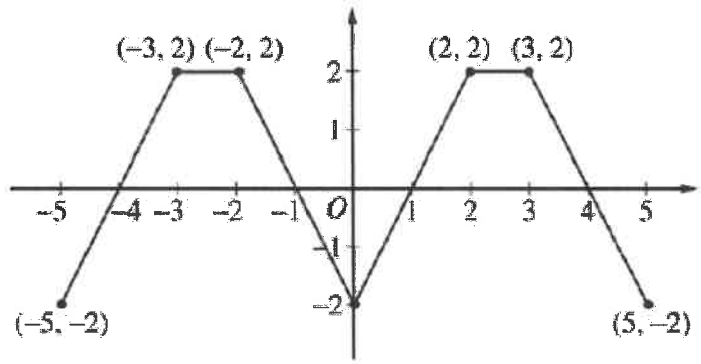
(c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.

(d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.

The graph of the function f shown above consists of six line segments. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

(a) Find $g(4)$, $g'(4)$, and $g''(4)$.

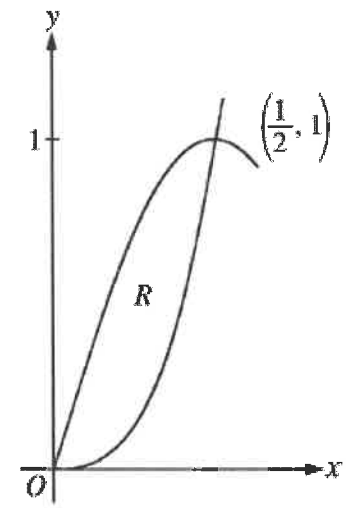
(b) Does g have a relative minimum, a relative maximum, or neither at $x = 1$? Justify your answer.



Graph of f

(c) Suppose that f is defined for all real numbers x and is periodic with a period of length 5. The graph above shows two periods of f. Given that $g(5) = 2$, find $g(10)$ and write an equation for the line tangent to the graph of g at $x = 108$.

2011 AP® CALCULUS AB
Question 3



Let R be the region in the first quadrant enclosed by the graphs of $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$, as shown in the figure above.

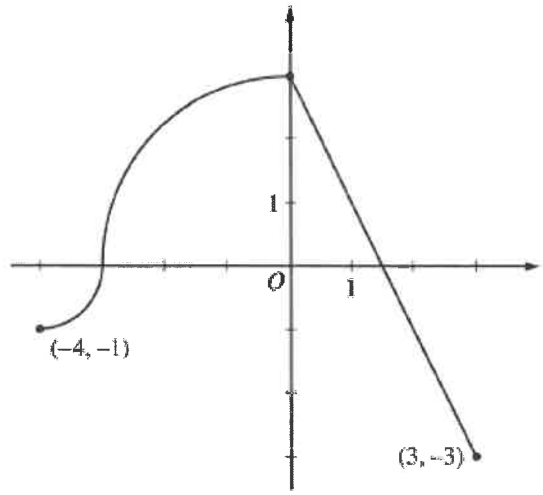
(a) Write an equation for the line tangent to the graph of f at $x = \frac{1}{2}$.

(b) Find the area of R.

(c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

2011 AP® CALCULUS AB
Question 4

The continuous function f is defined on the interval $-4 \le x \le 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure above.



Graph of f

Let $g(x) = 2x + \int_0^x f(t)\,dt$.

(a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.

(b) Determine the x-coordinate of the point at which g has an absolute maximum on the interval $-4 \le x \le 3$. Justify your answer.

(c) Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.

(d) Find the average rate of change of f on the interval $-4 \le x \le 3$. There is no point c, $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.